Appendix-A2

Sample calculation: **Cobalt Target**

i) **Calculation of correction for the intrinsic efficiency of the detector for incident energy** \( (\varepsilon_i) \):

The detector’s intrinsic efficiency for \( i = 32.86 \text{ keV} \) is calculated using the relation,

\[
\varepsilon_i = 1 - \exp(-\mu_{iGe} t_{Ge}) ; \mu_{iGe} = 6.93 \text{ cm}^2/\text{g} ; t_{Ge} = 5.32 \text{ g/cm}^2
\]

\( \varepsilon_i = 1.0 \)

ii) **Calculation of solid angle correction for the incident photons** \( (G_i) \):

Assuming the radioactive source to be a point source, the solid angle correction factor is calculated using the relation,

\[
G_i = 1 - \frac{d}{\sqrt{d^2 + r^2}} ; d = 5 \text{ mm} ; 12.7 \text{ mm}
\]

\( G_i = 0.63 \)

iii) **Calculation of window attenuation for incident photons**:

As explained in the experimental details in chapter 3, the window attenuation correction factor for the incident photons is calculated using the relation,

\[
\exp(-\mu_{iBe} t_{Be}) ; \mu_{iBe} = 0.174 \text{ cm}^2/\text{g} ; t_{Be} = 0.1109 \text{ g/cm}^2
\]

\( \exp(-\mu_{iBe} t_{Be}) = 0.98 \)

iv) **Calculation of self-attenuation of emitted K X-ray photons in the target** \( (\beta) \):

The self-attenuation correction \( \beta \) for the emitted K X-ray photons from the target element is calculated as in equation (2.6).

The calculation of self-attenuation correction factor for cobalt target element is shown below,
\[ \beta = \frac{1 - \exp(-\mu \cdot t)}{(\mu_i + \mu_e) \cdot t}; \mu_i = 6.93 \text{ cm}^2/\text{g}; \mu_e = 48.1 \text{ cm}^2/\text{g}; t = 0.0111 \text{ g/cm}^2. \]

\[ \beta = 0.75 \]

\( v \) Calculation of correction for the intrinsic efficiency of the detector for emitted K X-ray energy \( (\varepsilon_X) \):

The intrinsic efficiency of the detector for the emitted X-ray photons of given energy is calculated as in (Equation 2.7) of chapter 2. The calculation of intrinsic efficiency of the detector for cobalt K X-ray weighted average energy \( (X = 7.00 \text{ keV}) \) is as follows,

\[ \varepsilon_X = 1 - \exp(-\mu_{\text{XGe}} \cdot t_{\text{Ge}}); \mu_{\text{XGe}} = 99.1 \text{ cm}^2/\text{g}; t_{\text{Ge}} = 5.32 \text{ g/cm}^2 \]

\[ \varepsilon_X = 1.0 \]

\( vi \) Calculation of solid angle correction for the emitted photons \( (G_X) \):

The solid angle correction for the emitted K X-ray photons of X energy is calculated as in (Equation 2.10),

\[ G_X = \frac{1}{2} \left( 1 - \frac{d}{\sqrt{d^2 + r^2}} \right); d = 5 \text{ mm}; 12.7 \text{ mm} \]

\[ G_X = 0.32 \]

\( vii \) Calculation of window attenuation for K X-ray photons at X energy:

The window attenuation correction factor for the emitted K X-ray photons at X energy is calculated using the relation,

\[ \exp(-\mu_{\text{XBe}} t_{\text{Be}}); \mu_{\text{XBe}} = 1.62 \text{ cm}^2/\text{g}; t_{\text{Be}} = 0.1109 \text{ g/cm}^2 \]

\[ \exp(-\mu_{\text{XBe}} t_{\text{Be}}) = 0.84 \]

\( a \) Actual / true incident intensity \( (I_0) \):

From the background corrected source spectrum, the measured incident number of photons is \( I'_0 = 13164029 \text{ counts / 3hours} \)

The actual / true incident intensity is calculated as shown below,
\[ I_0 = \frac{I'_0}{\varepsilon_i G_i \exp(-\mu_{iBe} t_{Be})} \]; substituting the values of \( I'_0 \), \( \varepsilon_i \), \( G_i \) and \( \exp(-\mu_{iBe} t_{Be}) \),

\[ I_0 = \frac{13164029}{1 \times 0.63 \times 0.98} \]

\[ I_0 = 21321718 \text{ counts / 3hours} \]

b) Corrected K X-ray intensity (\( I_K \)):

Here, we present the estimation of corrected K X-ray intensity (\( I_K \)) for cobalt target element. From the K X-ray fluorescence spectrum, the measured K X-ray photons are 118151 counts / 3 hours.

Applying geometry correction, detectors’ intrinsic efficiency at 7.00 keV, self-attenuation and correction for window attenuation, the actual number of K X-ray photons in 4\( \pi \) sr is '\( I_K' \) is calculated using equation (2.5),

\[ I_K = \frac{I'_K}{\beta \varepsilon_X G_X \exp(-\mu_{XBe} t_{Be})} \]; substituting the values of \( I'_K \), \( \varepsilon_X \), \( G_X \) and \( \exp(-\mu_{XBe} t_{Be}) \),

\[ I_K = \frac{118151}{0.75 \times 1.0 \times 0.32 \times 0.84} = 586066 \]

The total number of K X-ray photons, \( I_K = 586066 \text{ counts / 3hours} \)

From the measured incident intensity and the emitted K X-ray intensity from cobalt target pure element, the K X-ray fluorescence parameters are determined using (equations in Chap. 2). The determination of these parameters is presented as below.

**Determination of K X-ray fluorescence parameters**

K X-ray fluorescence yield (\( \omega_K \)): From equation (2.4),
\[ \omega_K = \frac{I_K}{n_K} \text{; where,} \quad I_K \text{ is the total number of K X-ray photons and} \quad n_K \text{ is the number of K-shell vacancies calculated using the relation,} \]

\[ n_K = I_0 n_a \tau_K \]

\( I_0 \) is the total number of incident photons, \( n_a \) is the number of atoms per sq. cm of the target \( \tau_K \) is the K-shell photoelectric cross-section at 32.86 keV is (640 barn/atom) taken from WinXcom data. The number of atoms per sq.cm of the cobalt target is calculated by,

\[ n_a = \frac{N_A t}{A} \text{; where,} \quad N_A \text{ is the Avogadro’s number,} \quad t= \text{mass thickness in gm/cm}^2 \text{ and } A \text{ is the atomic weight of cobalt element. Substituting and simplifying,} \]

\[ n_a = \frac{6.022 \times 10^{23} \times 0.0111}{58.93} \text{ ; } n_a = 1.134 \times 10^{20} \text{ atoms/cm}^2 \]

The number of vacancies created \( n_K \) is,

\[ n_K = 21321718 \times 1.134 \times 10^{20} \times 640 \times 10^{-24} \]

\[ \therefore n_K = 1547445 \text{ vacancies} \]

Substituting for \( I_K \) and \( n_K \) in equation (2.4), \( \omega_K = \frac{586066}{1547445} \)

The K X-ray fluorescence yield for cobalt pure element is,

\[ \omega_K = 0.379 \]

**K\text{\textsubscript{a}} and K\text{\textsubscript{\beta}} fluorescence yields (\( \omega_{K\text{\textsubscript{a}}} \) and \( \omega_{K\text{\textsubscript{\beta}}} \)):** The fluorescence yields of the K\text{\textsubscript{a}} and K\text{\textsubscript{\beta}} X-ray emission lines are calculated using,

\[ \omega_{K\text{\textsubscript{\alpha}}} = \frac{l_{K\text{\textsubscript{\alpha}}}}{n_K} \text{ and } \omega_{K\text{\textsubscript{\beta}}} = \frac{l_{K\text{\textsubscript{\beta}}}}{n_K} \]

\[ \omega_{K\text{\textsubscript{\alpha}}} = \frac{534590}{1547445} \text{ and } \omega_{K\text{\textsubscript{\beta}}} = \frac{55077}{1547445} \]

\[ \omega_{K\text{\textsubscript{\alpha}}} = 0.345 \text{ and } \omega_{K\text{\textsubscript{\beta}}} = 0.036 \]
**K X-ray fluorescence cross-section ($\sigma_K$):** The K X-ray fluorescence cross-section at incident energy of 32.86 keV is calculated using the relation, $\sigma_K = \omega_K \tau_K$ substituting for ‘$\omega_K$’ from equation (2.4)

$$\sigma_K = \frac{l_K}{l_0 n_a} = \frac{586066}{21321718 \times 1.134 \times 10^{20}}$$

K X-ray fluorescence cross-section at 32.86 keV is,

$$\sigma_K = 242.4 \text{ b/atom}$$

**$K_\alpha$ and $K_\beta$ X-ray fluorescence cross-section at 32.86 keV ($\sigma_{K\alpha}$ and $\sigma_{K\beta}$):** These parameters are calculated using,

$$\sigma_{K\alpha} = \frac{l_{K\alpha}}{l_0 n_a} \text{ and } \sigma_{K\beta} = \frac{l_{K\beta}}{l_0 n_a};$$

$$\sigma_{K\alpha} = \frac{534590}{21321718 \times 1.134 \times 10^{20}}$$

$$\sigma_{K\beta} = \frac{55077}{21321718 \times 1.134 \times 10^{20}}$$

$$\sigma_{K\alpha} = 221.1 \text{ b/atom} \text{ and } \sigma_{K\beta} = 22.8 \text{ b/atom}$$

**Ratio between the widths of the radiative transition and the radiationless (Auger) transition for the K-shell ($R_K^K/R_A^K$):**

The ratio is calculated using the equation (2.17),

$$\frac{R_K^K}{R_A^K} = \frac{l_K}{(n_K-I_K)}; \text{ Substituting for ‘}l_K\text{’ and ‘}n_K\text{’}$$

$$\frac{R_K^K}{R_A^K} = \frac{586065}{(1547445-586065)},$$

$$\frac{R_K^K}{R_A^K} = 0.61$$
**K to L shell total vacancy transfer probability ($\eta_{KL}$):**

The K to L shell total vacancy transfer probability is estimated using the relation as given by Schönfeld and Janßen [56],

$$\eta_{KL} = \frac{2 - \omega_K}{1 + (I_{K\beta} / I_{K\alpha})}$$

Substituting for ‘$\omega_K$’ and ‘$(I_{K\beta} / I_{K\alpha})$’ in the above equation and simplifying,

$$\eta_{KL} = \frac{2 - 0.379}{1 + (0.122)}$$

The K to L shell total vacancy transfer probability for cobalt pure element is,

$$\eta_{KL} = 1.45$$

**K X-ray intensity ratio ($I_{K\beta} / I_{K\alpha}$):**

From the cobalt K X-ray fluorescence spectrum, The $K_{\alpha}$ X-ray photons from the fluorescence spectrum, $I_{K\alpha}' = 2624$ counts / 3 hours

Substituting and simplifying in, $I_{K\alpha} = \frac{I_{K\alpha}'}{Gx \epsilon_{K\alpha} \beta_{K\alpha} \exp(-\mu_{XBe} t_{Be})}$

$$I_{K\alpha} = \frac{2624}{0.32 \times 0.831 \times 0.695 \times 0.83}$$

∴ The total number of $K_{\alpha}$ X-ray photons is,

$$I_{K\alpha} = 17106 \text{ counts / 3 hours}$$

Similarly, The $K_{\beta}$ X-ray photons from the fluorescence spectrum, $I_{K\beta}' = 378$ counts / 3 hours

Substituting and simplifying, $I_{K\beta} = \frac{I_{K\beta}'}{Gx \epsilon_{K\beta} \beta_{K\beta} \exp(-\mu_{XBe} t_{Be})}$

$$I_{K\beta} = \frac{378}{0.32 \times 0.87 \times 0.75 \times 0.87}$$

∴ The total number of $K_{\beta}$ X-ray photons is,

$$I_{K\beta} = 2081 \text{ counts / 3 hours}$$
The K X-ray intensity ratio of cobalt element is,

\[
\left( \frac{I_{K\beta}}{I_{K\alpha}} \right) = \frac{I'_{K\beta} e^{K\beta \lambda_{K\alpha}} \exp(-\mu_{XBe} t_{Be})}{I'_{K\alpha} e^{K\beta \lambda_{K\beta}} \exp(-\mu_{XBe} t_{Be})} \\
\left( \frac{I_{K\beta}}{I_{K\alpha}} \right) = \frac{378 \times 0.831 \times 0.695 \times 0.83}{2624 \times 0.87 \times 0.75 \times 0.87} \\
\frac{I_{K\beta}}{I_{K\alpha}} = 0.122
\]

Total atomic attenuation cross-section for cobalt target at 32.86 keV (\(\sigma_t\)):

From,

\[
\sigma_t(32.86) = \frac{A}{N_A} \times \frac{\ln(I_t/I_0)}{t} \text{ b/atom}
\]

The number of incident photons in the transmitted spectrum,

\(I'_t = 12045086 \text{ counts / 3 hours}\)

\[
I_t = \frac{I'_t}{g_t G_t \exp(-\mu_{XBe} t_{Be})} \; ; \; I_t = \frac{12045086}{1 \times 0.63 \times 0.98} ;
\]

\(I_t = 19509371 \text{ counts / 3 hours}\)

Substituting for \(A, N_A, I_0, I_t\) and \(t\),

\[
\sigma_t(32.86) = \frac{58.93}{6.022 \times 10^{23}} \times \frac{\ln(19509371/21321781)}{0.0111} ; \quad \sigma_t(32.86) = 783 \text{ b/atom}
\]

K X-ray Jump factor \((J_K)\):

The K X-ray jump factor is calculated by the relation,

\[
J_K = \frac{\sigma_{K\alpha}(E)}{(\sigma_t - \sigma_{ts}) n_K} \left[ 1 + \left( \frac{I_{K\beta}}{I_{K\alpha}} \right) \right]
\]

where, ‘\(\sigma_{K\alpha}(E)\)’ is the K\(\alpha\) X-ray photons fluorescence cross-section and the total atomic scattering cross-section ‘\(\sigma_{ts}\)’ at the incident energy from WinXcom. ‘\(\sigma_{ts} = 37.5 \text{ b/atom}\)’

\[J_K = 0.87\]
K X-ray jump ratio ($r_K$):
From the calculated value of K X-ray jump factor, the K X-ray jump ratio is calculated as,

$$r_K = \frac{1}{1-J_K}$$

$$r_K = 7.7$$

Error estimation in the measurements of K X-ray fluorescence parameters
In this section, we discuss the uncertainties in the determined K X-ray fluorescence parameters for cobalt target. The calculation of weighted average value and the associated weighted error for $\omega_K$ values is estimated and presented briefly.

i) Uncertainty in K X-ray fluorescence yield ($\omega_K$);
The standard deviation in K X-ray fluorescence yield ‘$\omega_K$’ is calculated using,

$$\sigma_{\omega_K} = \omega_K \sqrt{\left(\frac{\sigma_{lK}}{l_K}\right)^2 + \left(\frac{\sigma_{nK}}{n_K}\right)^2}$$

$$\sigma_{\omega_K} = 0.379 \times \sqrt{\left(\frac{37858}{586065}\right)^2 + \left(\frac{15810}{1547445}\right)^2}$$

$$\sigma_{\omega_K} = 0.379 \times \sqrt{4.17 \times 10^{-3} + 1.04 \times 10^{-4}}$$

The uncertainty in the K X-ray fluorescence yield is, $\sigma_{\omega_K} = 0.025$

∴ The K X-ray fluorescence yield of cobalt target of trial 1 is,

$$\omega_K = 0.379 \pm 0.025$$

ii) Uncertainty in the $K_\alpha$ and $K_\beta$ fluorescence yield ($\omega_{K\alpha}$ and $\omega_{K\beta}$):
Similarly, the error in the $K_\alpha$ and $K_\beta$ fluorescence yield is calculated as,
\[
\sigma_{\omega_{K\alpha}} = \omega_{K\alpha} \sqrt{\left( \frac{\sigma_{l_{K\alpha}}}{l_{K\alpha}} \right)^2 + \left( \frac{\sigma_{n_{K}}}{n_{K}} \right)^2}
\]

\[
\sigma_{\omega_{K\beta}} = \omega_{K\beta} \sqrt{\left( \frac{\sigma_{l_{K\beta}}}{l_{K\beta}} \right)^2 + \left( \frac{\sigma_{n_{K}}}{n_{K}} \right)^2}
\]

The uncertainty is, \( \sigma_{\omega_{K\alpha}} = 0.001 \) and \( \sigma_{\omega_{K\beta}} = 0.001 \)

\( \therefore \) The \( \omega_{K\alpha} \) and \( \omega_{K\beta} \) of cobalt is,

\[
\omega_{K\alpha} = 0.345 \pm 0.001 \text{ and } \omega_{K\beta} = 0.036 \pm 0.001
\]

iii) Uncertainty in K X-ray fluorescence cross-section (\( \sigma_{K} \)):

The standard deviation in K X-ray fluorescence cross-section \( \sigma_{K} \) is calculated using, Let \( \sigma_{K} = \frac{l_{K}}{l_{0n_{a}}} \equiv F_{K} \)

\[
\sigma_{F_{K}} = F_{K} \sqrt{\left( \frac{\sigma_{l_{K}}}{l_{K}} \right)^2 + \left( \frac{\sigma_{l_{0}}}{l_{0}} \right)^2 + \left( \frac{\rho_{n_{a}}}{n_{a}} \right)^2}
\]

\[
\sigma_{F_{K}} = F_{K} \sqrt{\left( \frac{37858}{586065} \right)^2 + \left( \frac{21364}{21321718} \right)^2 + \left( \frac{1.134 \times 10^{18}}{1.134 \times 10^{20}} \right)^2}
\]

\[
\sigma_{F_{K}} = 242.4 \times 10^{-24} \sqrt{4.27 \times 10^{-3}} = 15.85 \text{ b/atom}
\]

\( \therefore \) The K X-ray fluorescence cross-section of cobalt target at 32.86 keV of trial 1 is, \( \sigma_{K} = 242.4 \pm 15.85 \text{ b/atom} \)

iv) Uncertainty in K\(_\alpha\) and K\(_\beta\) X-ray fluorescence cross-section of cobalt target at 32.86 keV (\( \sigma_{K\alpha} \) and \( \sigma_{K\beta} \)):

Let the error in these be denoted by, \( F_{\sigma_{K\alpha}} \) and \( F_{\sigma_{K\beta}} \).
The uncertainty is, \( F\sigma_{K\alpha} = 6.9 \text{ b/atom} \) and \( F\sigma_{K\beta} = 0.75 \text{ b/atom} \)

\[ √(\frac{F\sigma_{IK\alpha}}{l_{K\alpha}})^2 + (\frac{\sigma_{l_0}}{l_0})^2 + (\frac{\sigma_{n_0}}{n_0})^2 \]

\[ √(\frac{F\sigma_{IK\beta}}{l_{K\beta}})^2 + (\frac{\sigma_{l_0}}{l_0})^2 + (\frac{\sigma_{n_0}}{n_0})^2 \]

The uncertainty is found to be \( σ_{K\alpha} = 221.1 \pm 6.9 \text{ b/atom} \) and \( σ_{K\beta} = 22.8 \pm 0.75 \text{ b/atom} \)

v) Uncertainty in the ratio between the widths of the radiative transition and the radiationless (Auger) transition for the K-shell of cobalt target \( (\frac{Γ_{R}^{K}}{Γ_{A}^{K}}) \)

The uncertainty in \( (\frac{Γ_{R}^{K}}{Γ_{A}^{K}}) \) is calculated as,

\[ σ_{Γ_{R}^{K}/Γ_{A}^{K}} = (\frac{Γ_{R}^{K}}{Γ_{A}^{K}}) \sqrt{\left(\frac{σ_{l_0}^{K}}{l_0} \right)^2 + \left(\frac{σ_{n_0}^{K}}{n_0} \right)^2 + \left(\frac{σ_{l_0}^{K}}{n_0} - l_0 \right)^2} \]

The uncertainty is found to be \( σ_{Γ_{R}^{K}/Γ_{A}^{K}} = 0.05 \)

\[ Γ_{K}^{K} = 0.61 \pm 0.05 \]

vi) Uncertainty in K X-ray intensity ratio \( (I_{K\beta}/I_{K\alpha}) \):

The uncertainty present in the K X-ray intensity ratio is calculated as,

\[ σ_{(I_{K\beta}/I_{K\alpha})} = \sqrt{\left(\frac{σ_{I_{K\beta}}}{I_{K\beta}} \right)^2 + \left(\frac{σ_{I_{K\alpha}}}{I_{K\alpha}} \right)^2} \]
\[\sigma_{(\frac{I_{KB}}{I_{Ka}})} = 0.122\sqrt{\left(\frac{273}{17106}\right)^2 + \left(\frac{90}{2081}\right)^2}\]

The uncertainty in the K X-ray intensity ratio \(\sigma_{(\frac{I_{KB}}{I_{Ka}})} = 0.01\)

\[\therefore \text{The K X-ray intensity ratio of cobalt target of trial 1 is,}\]

\[
\frac{I_{KB}}{I_{Ka}} = 0.122 \pm 0.01
\]

vii) Uncertainty in the calculation of total vacancy transfer probability \(\eta_{KL}\):

\[
\sigma_{\eta_{KL}} = \eta_{KL} \sqrt{\left(\frac{\sigma_{\omega_K}}{\omega_K}\right)^2 + \left(\frac{\sigma_{\frac{I_{KB}}{I_{Ka}}}}{\frac{I_{KB}}{I_{Ka}}}\right)^2} = 1.45 \times \sqrt{\left(\frac{0.025}{0.379}\right)^2 + \left(\frac{0.01}{0.122}\right)^2}
\]

The uncertainty in the K to L shell total vacancy transfer probability \(\sigma_{\eta_{KL}} = 0.15\)

\[\therefore \text{The K to L shell total vacancy transfer probability for cobalt of trial 1 is,}\]

\[
\eta_{KL} = 1.45 \pm 0.15
\]

viii) Error factor in the calculation of the total atomic attenuation cross-section for cobalt target at 32.86 keV \(\sigma_t\):

Let \(F_t = \sigma_{t(32.86)} = \frac{A}{N_A} \times \frac{\ln(I_t/I_0)}{t} \text{ b/atom}\)

The uncertainty in the total atomic attenuation cross-section for cobalt target at 32.86 keV is calculated as,

\[
\sigma_{F_t} = A_1 \left[\sigma_{(\frac{I_t}{I_0})} + \sigma_t\right] \quad \text{Where, } A_1 = \frac{A}{N_A} \text{ is constant}\]
The uncertainty in total atomic attenuation cross-section for cobalt at 32.86 keV is, $\sigma_{F_t} = 0.012$ b/atom

\[\therefore \text{The total atomic attenuation cross-section for cobalt target at 32.86 keV is,} \quad \sigma_{t(32.86)} = 783 \pm 0.012 \text{ b/atom}\]

**ix) Uncertainty in K X-ray Jump factor ($J_K$):**

From equation (2.20),

Let, $\sigma_{\text{K}_\alpha}(E) \equiv F_{\text{K}_\alpha}(E)$ and $(\sigma_t - \sigma_{\text{t}_s}) \equiv F_{(X)}$, then the uncertainty in K X-ray jump factor is calculated as,

\[
\sigma_{J_K} = J_K \left[ \left( \frac{\sigma_{F_{\text{K}_\alpha}(E)}}{F_{\text{K}_\alpha}(E)} \right)^2 + \left( \frac{\sigma_{F_{(X)}}}{F_{(X)}} \right)^2 + \left( \frac{\sigma_{\omega_K}}{\omega_K} \right)^2 + \left( \frac{\sigma_{I_{K}\beta}}{I_{K}\alpha} \right)^2 \right]^{1/2}
\]

Substituting

The uncertainty in the K shell jump factor for cobalt target is $\sigma_{J_K} = 0.067$

\[\therefore \text{The K shell jump factor for cobalt element is,} \quad J_K = 0.87 \pm 0.067\]

**x) Uncertainty in K X-ray Jump ratio ($r_K$):**

The uncertainty in the jump ratio is estimated as, $\sigma_{r_K} = r_K \left[ \left( \frac{\sigma_{J_K}}{J_K} \right)^2 \right]^{1/2}$

The uncertainty in the jump ratio for cobalt target is estimated as, $\sigma_{r_K} = 0.4$

\[\therefore \text{The jump ratio of cobalt pure element is,} \quad r_K = 7.7 \pm 0.6\]

The procedure is repeated and 4 trials is obtained for cobalt target. For every trial K X-ray fluorescence parameters and the error/uncertainty
associated with each parameter are calculated independently. From the
determined values of K X-ray fluorescence parameter of all the 4 trials
for a parameter,

The weighted average and the error associated is calculated using the
equations, \( \langle x \rangle = \frac{\sum_{i=1}^{N} a_i x_i}{\sum_{i=1}^{N} a_i} \)

The weighted factor in the uncertainty is calculated using,
\[
\sigma_{\langle x \rangle} = \frac{1}{\sqrt{\sum_{i=1}^{N} \left(1/\sigma_{x_i}^2\right)}}
\]

The weighted average value and the associated weighted error from 4
trials for, **fluorescence yield** \( (\omega_K) \) is,

\[
\langle \omega_K \rangle = \frac{\sum_{i=1}^{4} \omega_{K_i} x_i}{\sum_{i=1}^{4} \omega_{K_i}}
\]

\[
= \frac{(0.379 \times 0.025) + (0.378 \times 0.026) + (0.379 \times 0.025) + (0.379 \times 0.024)}{(0.025 + 0.026 + 0.025 + 0.024)}
\]

Hence the weighted average value of fluorescence yield \( \langle \omega_K \rangle = 0.379 \)

Now the weighted error associated in the estimation of fluorescence yield
of lead pure element is calculated as,

\[
\sigma_{\langle \omega_K \rangle} = \frac{1}{\sqrt{\frac{1}{(0.025)^2} + \frac{1}{(0.026)^2} + \frac{1}{(0.025)^2} + \frac{1}{(0.024)^2}}}
\]

\( ; \sigma_{\langle \omega_K \rangle} = 0.012 \)

From the trials, determination of fluorescence yield of cobalt pure
element according to the present study is,

\( \omega_K = (0.379 \pm 0.012) \)