CHAPTER I
INTRODUCTION

1.1 Objective and Scope

Convective flow and heat transfer has gained growing interest. This fact has been motivated by its importance in many engineering applications such as building thermal insulation, geothermal systems, food processing and grain storage, solar power collectors, contaminant transport in groundwater, casting in manufacturing processes, drying processes, nuclear waste, just to name a few.

Effect of buoyancy or surface tension can become a major mechanism of driving a possible convective instability for a horizontal fluid layer heated from below and cooled from above. The instability of convection driven by buoyancy is referred to as the Rayleigh-Bénard convection and the instability convection driven by surface tension is referred to as the Marangoni convection while the instability due to the combined effects of the thermal buoyancy and surface tension is called the Bénard-Marangoni convection. Theoretical analysis of Marangoni-Bénard convection was started with the linear analysis by Pearson [1958] who assumed an infinite fluid layer, non deformable case and zero gravity in the case of no-slip boundary conditions at the bottom. He showed that thermocapillary forces could cause convection when the Marangoni number exceeds a critical value in the absence of buoyancy forces. Pearson [1958] obtained the critical Marangoni number, $M_c = 79.607$ and the critical wave number $a_c = 1.99$: In the above Marangoni- Bénard instability analysis, the convective instability is induced by the temperature gradient which is decreasing linearly with fluid layer height.
The thermocapillary driven flow are important in many industrial processes which involve a fluid with a free surface, for example, several popular crystal growing techniques and coating processes. Internal heat generation also plays a significant role in many practical situations, such as Joule heating due to the flow of an electric current through a conducting fluid, the radiative heating and cooling of molten glass and heating of water in a solar collector. In some of these situations, for example the processing of molten glass, both effects are present. Sparrow et al. [1964] and Roberts [1967] analyze the thermal instability in a horizontal fluid layer with the nonlinear temperature distribution which is created by an internal heat generation. The effect of a quadratic basic state temperature profile caused by internal heat generation was first addressed by Char and Chiang [1994] for Bénard-Marangoni convection. Later, Wilson [1997] investigated the effect of the internal heat generation on the onset of Marangoni-Bénard convection when the lower boundary is conducting and when it is insulating to temperature perturbations. He found that the effect of increasing the internal heat generation is always to destabilize the layer. Shivakumara and Suma (2000) have investigated the effect of through flow and internal heat generation on the onset of convection using rigid and perfectly conducting boundaries. Recently Mokhtar et al (2003) have studied the effect of internal heat generation on the onset of Marangoni convection in porous media.

The non-Newtonian fluids (e.g. polymeric suspensions, animal blood, liquid crystals) which have very small-sized suspended particles of different shapes. These particles may change shape, may shrink and expand, and moreover, they may execute rotation independent of the rotation and movement of the fluid. Due to the addition of particles the classical Navier-
Stokes theory had to be reexamined. Most practical problems involving these types of working fluids are non-isothermal and these thermally responding fluids have uncovered new application areas. ‘Convection’ is a dominant and important mode of heat transport in many such applications and hence needs to be looked into. The theory of micropolar fluid is due to Eringen (1966), whose theory allows for the presence of particles in the fluid by additionally accounting for particle motion. The motivation for the study comes from many applications involving unclean fluids wherein the clean fluid is evenly interspersed with particles, which may be dust, dirt, ice or raindrops, or other additives (see Eringen(1999), Lukaszewicz (1999)). This suggests geophysical or industrial convection contexts for the application of micropolar fluids. Many authors have investigated the problem of Rayleigh-Bénard convection in Eringen’s micropolar fluid and concluded that the stationary convection is the preferred mode.

Some of the important applications concerning the problems studied in this dissertation are:

1. APPLICATION INVOLVING SUSPENSIONS

The aforementioned applications, as we have seen, involve two phases: liquid phase and solid phase. The solid phases essentially are static and allow passage of the liquids through its pores.

In situations where the grain sizes are very very small, the fluid dynamics, due to whatever reason, may carry away the grains. The grains may also execute motions relative to the carrier fluid. The dynamics in such suspensions is also of practical interest (see Lukaszewicz 1999, Eringen 1999 and Power 1995).
2. GEOPHYSICS

The Rayleigh-Benard convection has been extensively used to study the motion in the earth’s interior and to understand the mechanism of transfer of energy from the deep interior of the earth to the shallow depths (see Turcotte and Oxburgh 1972). This transfer of energy plays an important role in geothermal activities.

3. PLANETARY AND STELLAR CONVECTION

In astrophysics, the Rayleigh-Benard convection has been extensively studied to understand the formation of photo-stars, dynamo principle and so on. This convection helps to reduce the uncertainties that exist in other physical processes in stars and permits a better empirical determination of the arbitrary parameters used in stellar convection with other processes of stellar fluid dynamics which brings new instabilities is also one of the most interesting problems (Rudraiah et al 1985 a,b).

The important problems here are:

(i) to know when convection stabilizes or destabilizes pulsation,
(ii) to know the role of convection in the rotational history of the sun and
(iii) to know the effect of cosmic dust on solar convection.

With the motivation directed by the above applications, the main objective of this dissertation is to study the linear Rayleigh- Bénard and Rayleigh – Bénard - Marangoni convection in a micropolar fluids in the presence of external constraints magnetic field and internal heat generation. The scope of this dissertation lies in interpreting and explaining the mechanism of augmenting or suppressing convection. Having discussed
about the applications, motivations, objective and scope of the problems investigated in the thesis, we now review literature pertaining to unconstrained / constrained Rayleigh- Bénard convective instabilities in Newtonian and micropolar fluids.
Chapter II
LITERATURE REVIEW

2.1 Rayleigh – Benard convection

Thomson (1882) briefly reported an experiment on the thermal instability phenomenon. Benard (1901) later presented a much more complete description of the development of the convective flow. Lord Rayleigh (1916) was the first person to give an analytical treatment of the problem aimed at determining the conditions delineating the breakdown of the basic state. As a consequence of these works the thermal instability problem is now called as Rayleigh-Benard convection. The determination of the condition for the onset of convection, which depends on the magnitude of the temperature difference, is expressed in dimensionless form as the critical value of the Rayleigh number. The Rayleigh theory was generalised and also extended to other boundary combinations by Jeffrey (1926, 1928) and Low (1929). Pellew and Southwell (1940) presented the most complete theory of the thermal instability problem. Chandrasekhar (1961) explained in detail, using linear theory based on an infinitesimal perturbations, the critical Rayleigh number (critical temperature gradient) at which the quiescent state breaks down for the case of a uniform temperature gradient. In view of this, a number of theoretical and experimental studies have been made that present various facts of thermal convection.

Thomson (1951) was the first person to examine the modifications produced in the Rayleigh (1916)-Jeffrey (1926) theory of slow thermal convection by the addition of Lorentz force caused by the interaction of magnetic field and conducting fluid. Walen (1944), Fermi (1949) and Alfven (1950) considered the interaction between the magnetic field and
electrically conducting fluids. Many authors (Chandrasekhar 1961, Singh and Cowling 1963, Riley 1964 and D’sa 1971) have carried out further investigations on Rayleigh-Benard convection in the presence of vertical magnetic field, called magnetoconvection.

Rudraiah (1981) has studied the three-dimensional steady linear and non-linear magnetoconvection and found that finite amplitude steady convection persists only for certain range of Chandrasekhar number and the hexagonal planform is the preferred cell pattern in the description of three-dimensional motion. In particular he has shown that the convective heat transport in hydrogen is much greater than that in mercury.

Weiss (1981a,b) has studied the two-dimensional non-linear magnetoconvection in a Boussinesq fluid in a series of numerical experiments and he has shown that the form of nonlinear solutions depend upon the competition between the thermally driven flow and the counter flow driven by Lorentz force in the flux sheet.

Rudraiah et al (1985a,b) studied two – and three – dimensional non-linear, magnetoconvection in a Boussinesq fluid in the presence of a vertical magnetic field.

Rudraiah and Chandna (1985) considered the effects of Coriolis force and non-uniform temperature gradient on the onset of Rayleigh-Benard convection in a thin, horizontal, rotating fluid layer using linear-stability analysis and the single-term Galerkin method. They also showed that a suitable strength of Coriolis force together with a non-uniform temperature gradient provide a mechanism for surpressing or augmenting convection which is important in the transfer of heat from one region to another.

The problem of free convection in a porous medium heated from below using the linearised system of equations has been studied by Muskat

The literature reviewed so far is concerned with Rayleigh-Benard convection with linear basic temperature profile. Now we discuss briefly the literature concerned with Rayleigh-Benard convection with non-linear basic temperature profile.

The nonlinearity of the basic temperature profile is due to rapid heating or cooling at a boundary. Graham (1933), Chandra (1938), Sutton (1950), Morton (1957) and Goldstein (1959) have investigated the thermal instabilities in the presence of non-linear basic temperature profiles.

George K. Perekattu and C. Balaji (1960) studied the results of an analytical and numerical investigation to determine the effect of internal heat generation on the onset of convection, in a differentially heated shallow fluid layer. The case with the bottom plate at a temperature higher than the top plate mimics the classical Rayleigh Benard convection.

D. J. Tritton(1979) and M. N. Zarraga (1967) studied the Linear stability analysis is first carried out for the case of an infinitely wide cavity. The effect of aspect ratio on the onset of convection is studied by solving the
full Navier–Stokes equations and the equation of energy and observing the
temperature contours. A bisection algorithm is used for an accurate
prediction of the onset. The numerical results are used to plot the stability
curves for eight different aspect ratios. A general correlation is developed to
determine the onset of convection in a differentially heated cavity for
various aspect ratios. For an aspect ratio of 10, it is seen that the cavity
approaches the limit of an infinite cavity. Analytical results obtained by
using linear stability analysis agree very well with the “full” CFD
simulations, for the above aspect ratio.

P. H. Roberts has theoretical studied an experiment by Tritton &
Zarraga (1967) in which convective motions were generated in a horizontal
layer of water (cooled from above) by the application of uniform heating.
The marginal stability problem for such a layer is solved, and a critical
Rayleigh number of 2772 is obtained, at which patterns of wave-number
2.63 times the reciprocal depth of the layer are marginally stable. The
remainder of the paper is devoted to the finite amplitude convection which
ensues when the Rayleigh number, $R$, exceeds 2772. The theory is
approximate, the basic simplification being that, to an adequate
approximation, Fourier decompositions of the convective motions in the
horizontal ($x, y$) directions can be represented by their dominant (planform)
terms alone. He discussion is given of this hypothesis, with illustrations
drawn from the (better studied) Bénard situation of convection in a layer
heated below, cooled from above, and containing no heat sources. The
hypothesis is then used to obtain ‘mean-field equations’ for the convection.
These admit solutions of at least three distinct forms: rolls, hexagons with
upward flow at their centres, and hexagons with downward flow at their
centres. Using the hypothesis again, the stability of these three solutions is
examined. It is shown that, for all $R$, a (neutrally) stable form of convection exists in the form of rolls. The wave-number of this pattern increases gradually with $R$. This solution is, in all respects, independent of Prandtl number. It is found, numerically, that the hexagons with upward motions in their centres are unstable, but that the hexagons with downward motions at their centres are completely stable, provided $R$ exceeds a critical value (which depends on Pandtl number, $P$, and which for water is about $3R_c$), and provided the wave-number of the pattern lies within certain limits dependent on $R$ and $P$.

Z. Hashim (1967) have studied experimental investigation has been made of cellular convection patterns produced by the instability of a horizontal layer of fluid heated uniformly throughout its body and cooled from above. The fluid was aqueous zinc sulphate solution, and the heating was produced by an electrolytic current. The flow was visualized and photographed using polystyrene beads which, because of a differential expansion, came out of suspension in regions where the fluid was hotter or colder than average.

D. J. Tritton and M. N Zarraga (1967) have studied the development of the cellular patterns as the Rayleigh number was varied was in many respects similar to that for Bénard convection. There were two striking differences. First, the fluid descended in the centres of the cells and ascended at the peripheries; this was probably associated with the asymmetry between the heating and cooling. Secondly, and more surprisingly, the horizontal scale of the convection patterns was unusually large (except at the lowest Rayleigh numbers); the distance between rising and falling currents was observed to reach typically five times the depth of the layer. This last observation may be relevant to theories of convection.
within the Earth's mantle, to mesoscale convection in the atmosphere, and to patterns formed in ice.

Nield (1975) discussed the onset of convection in the presence of non-uniform basic temperature gradient, considering only the constant heat flux condition at the boundaries. Rudraiah et al (1980,1982) have considered both constant heat flux and fixed surface temperature (isothermal) conditions at the boundaries and studied the effect of non-uniform temperature gradient on the onset of convection in a fluid saturated porous medium.

S. P. Bhattacharyya and S. K. Jena (1983) critical study on the stability of a hot layer of micropolar fluid heated from below with free boundaries has been investigated. The analysis shows that the method by which the previous investigators (Datta and Sastry, and Pérez-García and Rubí) obtained the critical Rayleigh number is not justified and the final result obtained thereby is erroneous. The correct solution to the problem has been presented. Moreover, it is found that the possibility of having an overstable marginal state which was shown by one of the previous investigators (Pérez-García and Rubí) is not justified. The correct approach proves the validity of the principle of exchange of stabilities for this problem. The results show that the criteria of micropolar stability have some interesting features having no classical analogue.

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S. P Bhattacharyya and M. Abbas (1985) have studied convective stability of a horizontal layer of incompressible micropolar fluid heated from below and rotating about a vertical axis has been investigated on the basis of linear theory, using normal mode analysis. The boundaries are assumed to be free. After introducing the corrections to the basic equations considered by Sastry and Rao [1], it has been found that the rotation has a destabilizing effect which contradicts the earlier assertion presented in [1]. Moreover, microinertia, which does not affect the stability of a hot horizontal layer of incompressible micropolar fluid in the absence of rotation [2], is found to have destabilizing effect.

K Jena and S. P. Bhattacharyya (1986) studied effect of microstructure on the thermal convection in a rectangular box of fluid heated from below has been investigated by applying the micropolar fluid theory. The influence of lateral walls on the convection process in a rectangular box has been determined. The Galerkin method has been employed to get an approximate solution for the eigenvalue problem. The beam functions which satisfy two boundary conditions on each rigid surface have been used to construct the finite roll (cells with two nonzero velocity components depend on all three spatial variables) trial functions for the Galerkin method. The effect of variations of material parameters at the onset of convection has been presented graphically. They observed that as the distance between the lateral
walls increases the effect of one of the material parameters, characterizing the spin-gradient viscosity, at the onset of stability diminishes. A comparison has been made with the corresponding results for a Newtonian fluid.

A. Khalili and I. S. Shivakumara (1988) have studied the Onset of convection in a horizontal porous layer is investigated including the effects of through-flow and a uniformly distributed internal heat generation for different types of hydrodynamic boundary conditions. The instability parameter is either a Darcy-external or Darcy-internal Rayleigh number which has been determined numerically using Galerkin technique. When the boundary conditions at the top and bottom are not identical, they is found that a small amount of through-flow in one particular direction destabilizes the system in the absence of heat generation. However, in the presence of an internal heat generation, through-flow in one direction destabilizes the system even if the boundary conditions at the two boundaries are of the same type and the destabilization is found to be more with the increase in internal Rayleigh number.

K. Julien (1996) has studied numerical simulation of thermal convection induced by internal heating in a three-dimensional thin flat box is performed. The upper boundary is held at a constant temperature and the lower boundary is thermally insulated with a uniform internal heating in a fluid layer. He observed hexagonal cells with downwelling at their center moderate Rayleigh number \( (Ra_H \leq 30Ra_H C) \). As the Rayleigh number increases, the cell size increases. He also observed a pattern transition: sheet-like downwellings gradually develop around the columnar downwelling with increasing Rayleigh number. At \( 30Ra_H C \) hexagonal cells with sheet downwelling at their center (spoke pattern) is observed. These sheets can be
seen only near the upper boundary. At $30Ra_{\mu}C \sim 70Ra_{\mu}C$, the cells experience successive transitions to the pattern which are dominated by sinking sheet.

Morten Tveitereid (1997) have studied the Steady solutions in the form of hexagons and two-dimensional rolls are obtained for convection in a horizontal porous layer heated from within. The stability of the flows with respect to small disturbances is investigated. They found that down-hexagons are stable for Rayleigh numbers $R$ up to 8 times the critical value ($8R_{c}$), while up-hexagons are unstable for all values of $R$. Moreover, two-dimensional rolls are found to be stable in the range $3R_{c}, < R < 7R_{c}$. Good agreement with some of the experimental observations of Buretta is found.

Tsan-Hui Hsu, Pao-tung Hsu and so-Tenn Tsail (1997) have discussed the Numerical study of natural convection flow in a tilting enclosure filled with micropolar fluids has been investigated. The enclosure is equipped with a single or multiple uniform heat sources. A two-dimensional, steady and laminar flow model is simulated. The heat transfer characteristic and flow phenomenon are presented for range values of the controlled parameters and situations, such as $Ra$ number, tilting angle of the enclosure and various material parameters of the fluid. The results indicate that dependence of microrotation term and heat transfer on microstructure parameters is significant. The effects of heat source locations and arrangement of multiple heat sources are studied to arrive at qualitative suggestions that may improve the cooling design of the system. The effect of the microrotation boundary conditions on heat transfer is discussed as well.

Oddmund Kvernvold (1977) studied the numerical analysis of convective motion in a model with one free and one rigid boundary is
performed. The stationary two-dimensional solutions are found and their dependence on the Rayleigh number, the wave number and the Prandtl number is discussed. The stability of the two-dimensional rolls with respect to three-dimensional disturbances is analysed. They found that the different disturbances depend strongly on the Prandtl number.

Abderrahim Mrabti, Kamal Gueraoui, Abdelaziz Hihi and Omar Terhmina (2000) have studied Numerical study of natural convection flow in a vertical cylindrical enclosure filled with micropolar fluids and heated from below has been investigated. A two-dimensional, steady and laminar flow model is simulated. The transport equations for vorticity, angular momentum and energy as well as the stream function equation are solved with the aid of the finite-difference method. The results indicate that dependence of microrotation term and heat transfer on microstructure parameters is significant. The heat transfer rate of micropolar fluids is found to be smaller than that of the Newtonian fluid.

2.2 Rayleigh – Benard – Marangoni (R – B – M) Convection With Classical Fourier Law

Rayleigh (1916) has explained the cells observed by Benard (1901) when a horizontal layer of fluid is heated from below in terms of buoyancy and Pearson (1958) in terms of surface tension. Nield (1964) has combined these rival theories. Fourier series method is used to obtain the eigenvalue equation for the case where lower boundary surface is a rigid conductor and the upper free surface is subjected to a general thermal condition. He has
found that the two agencies causing instability complement one another and are tightly coupled as both depend linearly on temperature gradient.

Nield (1966) has studied the effect of vertical magnetic field in an electrically conducting fluid on the onset of R-B-M convection. He has found that as the magnetic field strength increases, the coupling between the two agencies causing instability becomes weaker, so that the values of Rayleigh number at which convection begins tend to become independent of surface tension effects and similarly the critical Marangoni number tends to be unaffected by the buoyancy forces provided that the Rayleigh number is less than critical.

Namikawa and Takashima (1970) have studied the effect of uniform rotation on the onset of thermal instability in a horizontal layer of fluid by means of a linear stability analysis, assuming that one of the bounding surfaces is free and the other rigid. Fourier series method is used to obtain the eigenvalue. They have found that the Coriolis force has an inhibiting effect on the onset of convection even if the surface tension effect is taken into consideration, and as the speed of rotation increases the coupling between the two agencies causing instability becomes weaker.

Nield (1975) has investigated the onset of R-B-M convection in a horizontal fluid layer using non-uniform temperature gradients. The case of constant thermal flux boundary condition has been examined using a single-term Galerkin expansion method that gives quick and reasonable results and general basic temperature profiles are also treated. Some general conclusions about destabilising effects with respect to disturbances of infinitely long wavelengths of various basic temperature profiles are presented.
Lebon and Cloot (1981) have studied the effects of non-uniform temperature gradients on the onset of R-B-M convection in a horizontal fluid layer. Numerical calculations have been performed for three different temperature profiles corresponding respectively to a layer heated from below and cooled from above and a superposition of two layers at different temperatures. They have found that non-uniform temperature gradients promote instability; subcritical instabilities are also displayed.

Lebon and Cloot (1984) have studied the non-linear analysis of R-B-M convection in a horizontal fluid layer of infinite extent. Using the Gorkov-Malkus-Veronis technique, which consists of developing the steady solution in terms of a small parameter measuring the deviation from the marginal state, solves the non-linear equations describing the fields of temperature and velocity.

Friedrich, R. and Rudraiah, N.(1984) have investigated the onset of Marangoni convection driven by surface tension gradients in a thin horizontal fluid layer is studied by means of linear stability analysis, assuming that one of the bounding surfaces is free and adiabatic and the other rigid adiabatic or isothermal. A Galerkin technique is used to obtain the eigenvalues which are then computed numerically. The Coriolis force and inverted parabolic basic temperature profile are suitable for material processing in a microgravity environment, for they suppress Marangoni convection considerably.

Rudraiah et al (1986) have investigated the effect of non-uniform temperature gradient and a uniform vertical magnetic field on the onset of R-B-M magnetoconvection. The Galerkin method is used to obtain the eigenvalues, and a mechanism for suppressing and augmenting convection is discussed. They have found that with increase in magnetic field strength,
coupling between the two agencies becomes weaker even in the presence of non-uniform temperature gradients.

Rudraiah and Ramachandramurthy (1986) have studied the effects of non-uniform temperature gradient and the Coriolis force on the onset of convection driven by combined surface tension and buoyancy force by means of a linear stability analysis using Galerkin technique. A mechanism for suppressing or augmenting convection is discussed in detail.

Rudraiah et al (1986) have studied both linear and non-linear magnetoconvection in a rotating fluid layer. The analysis concentrated on the study of bifurcation and the destabilizing nature of rotation and magnetic field acting together. Theory of self-adjoint operators is used to obtain important bounds of physical parameters. They have found that the heat transport of square cell in the non-linear theory approaches to that of rolls for large values of Taylor and Chandrasekhar numbers and a physical explanation has been given.

Wilson (1993 b) has studied the effect of vertical magnetic field on the onset of steady R-B-M convection in a horizontal layer of quiescent, electrically conducting fluid subject to a uniform vertical temperature gradient. He has found that the critical Rayleigh and Marangoni numbers are dependent critically on the non-dimensional crispation and Bond numbers. The stability of the layer to long wavelength disturbances is analyzed and the two different asymptotic limits of strong surface tension and strong magnetic field are investigated. He has concluded that the presence of a magnetic field always has a stabilising effect.

Ming-I and Ko-Ta Chiang (1994) have investigated the effect of uniform distribution of internal heat generation on the stability of the Benard-Marangoni convection in a horizontal fluid layer with deformable
upper free surface. Using the fourth-order Runge-Kutta-Gill method with shooting technique, they have obtained the eigenvalues. The results indicate that the stability of R-B-M convection is significantly affected by internal heat generation in a fluid layer and by surface tension at the upper free surface.

Pai-Chuan Liu (1995) have studied the criteria for the onset of natural convection in a rotating liquid layer with nonuniform volumetric energy sources from absorbed thermal radiation are determined via linear stability analysis. The linearized perturbation equations are solved by using a numerical technique to obtain the eigenvalues that governs the onset of convection in a microgravity environment. The stability criteria are obtained in terms of the Marangoni number as function of the optical thickness. The influences of the Rayleigh number, Taylor number, Bond number, Crispation number, and Biot number on convection are examined in detail. These parameters provide a relationship between the critical Marangoni number and the Coriolis force, the buoyancy force, the interfacial tension, and the heat transport mechanisms.

I. S. Shivakumara, M. Venkatachalappa and S. P. Suma (1998) have investigated the onset of Marangoni convection with through flow in a horizontal fluid layer with upper boundary free and insulating to temperature perturbations and the lower boundary rigid and either conducting or insulating to temperature perturbations is investigated. The resulting eigenvalue problem is solved exactly. The Prandtl number arising due to throughflow plays a crucial role in determining the stability of the system. It is found that a small amount of throughflow in one particular direction destabilizes the system depending on the Prandtl number and temperature boundary conditions. Hashim and Wilson (1999c) have investigated the
R-B-M convection in a planar horizontal layer of fluid heated from below in the most physically relevant case when the non-dimensional Rayleigh and Marangoni numbers are linearly dependent. The comprehensive asymptotic analysis of the marginal curves in the limit of both long and short wavelength disturbances are studied.

El-Amin, M. F. (2001) has studied Magnetohydrodynamic free convection and mass transfer flow in micropolar fluid with constant suction. An analysis is presented for the problem of free convection with mass transfer flow for a micropolar fluid bounded by a vertical infinite surface under the action of a transverse magnetic field. Approximate solutions of the coupled nonlinear governing equations are obtained for different values of the microrotation – and the magnetic – parameters.

A. C. Or and R. E. Kelly (2001) have studied the feedback control of weakly nonlinear Rayleigh–Benard–Marangoni convection. The effect of proportional feedback control on the onset and development of finite-wavelength Rayleigh–Benard–Marangoni (RBM) convection using weakly nonlinear theory as applied to Nield's model, which includes both thermocapillarity and buoyancy but ignores deformation of the free surface. A two-layer model configuration is used, which has a purely conducting gas layer on top of the liquid. In the feedback control analysis, a control action in the form of temperature or heat flux is considered. Both measurement and control action are assumed to be continuous in space and time.

Y. N. Murty and V. V. Ramana Rao (2005) have studied the effect of throughflow onset of Marangoni convection in a horizontal layer of micropolar fluids flow bounded below by a rigid isothermal surface and above by a nondeformable free adiabatic surface, for marginal state. The
determination of the critical Marangoni number entails solving the eigenvalue problem numerically for the single-term Galerkin method is employed.

Y. Narasimha Murty(2005) has studied the effect of throughflow and uniform magnetic field on the onset of Marangoni convection in a horizontal layer of micro-polar fluids bounded below by a rigid isothermal surface and above by a non-deformable free adiabatic surface, for marginal state is studied. The conditions for the onset of instability occurring via stationary convection modes are obtained with the help of Galerkin method. It is observed that both stabilizing and destabilizing factors due to constant vertical throughflow can be enhanced by magnetic field.

Norihan Md. Arifin and Haliza Rosali (2007) have investigated on competition between modes of the onset of Marangoni convection with free-slip bottom under magnetic field. They used a numerical technique to analyze the onset of Marangoni convection in a horizontal layer of electrically-conducting fluid heated from below and cooled from above in the presence of a uniform vertical magnetic field. The top surface of a fluid is deformably free and the bottom boundary is rigid and free-slip. The critical values of the Marangoni numbers for the onset of Marangoni convection are calculated and the latter is found to be critically dependent on the Hartmann, Crispation and Bond numbers. In particular they presented an example of a situation in which there is competition between modes at the onset of convection.

Z. Siri and I. Hashim (2008) have studied the effect of feedback control on the onset of steady and oscillatory surface-tension-driven (Marangoni) convection in a rotating horizontal fluid layer with a flat free upper surface
and heated from below is considered theoretically using linear stability theory. The role of the controller gain parameter on the Ta–Pr parameter space, dividing stability domains into which either steady or oscillatory convection is preferred, is determined.

2.3 Rayleigh-Benard-Marangoni convection with Internal Heat source

Convection due to buoyancy or surface tension or combined buoyancy and surface tension force in which the fluid temperature decreases linearly with height. In other words, the heat transport in the quiescent state is purely by conduction. However, in many practical situations like the extraction and utilization of geothermal energy, nuclear reactor, subterranean porous layers and in the application of material sciences in space, it is of interest to determine the instability would be affected if the quiescent state was characterized by a non linear temperature profile. If there is internal heat generation within the fluid due to heat source or due to rapid heating or cooling at the boundaries. In such situations, instead of heating just from below, considering heat generation through the upper surface, so that the stratification is again unstable. Tritton and Zarraga (1967) investigated experimentally the effect of internal heat generation on convection where the motion is due to instability rather than due to the absences of an equilibrium configuration. Two striking results emerged from their experiments. First, the cell structure was, for moderate Rayleigh number, predominantly hexagonal with motion downwards at the center of each cell. Secondally, the horizontal scale of the convection pattern grew larger as the Rayleigh number was increased above its critical value. Roberts (1967) answered these experimental challenges using the nonlinear theory and has thrown more
light on the advantages and limitations of his approximate theory of finite-amplitude convection.

A nonlinear temperature distribution also arises due to rapid heating or cooling at a boundary. The effect of nonlinear basic temperature distribution on surface tension driven convection was analysed by Nield (1975).

J. H. Van Sant (1969) studied the maximum steady-state temperatures of freely convecting heat-generating fluids were measured experimentally. Two liquids of widely differing Prandtl and Grashof numbers were tested at many levels of internal heat generation rate in a horizontal pipe at constant wall temperature. A simplified analysis of buoyancy and viscous forces is given to show the method of correlating the temperature data. Experimental results have good correlation when presented according to the analysis.

N. Riahi (1984) has studied the problem of thermal convection is studied in a horizontal layer of fluid with an internal heat source which is restricted to vary linearly with the vertical variable. Square and hexagonal cells are found to be the only possible stable convection cells. Finite amplitude instability can occur for a range of the amplitude of convection. The presence of internal heating is found to be able to affect strongly the cell's size, the critical Rayleigh number, the heat transported by convection, the stability of the convective motion and the internal motion of the hexagonal cells.

Morten Tveitereid and Enok Palm (1976) have studied convection generated by uniformly distributed internal heat sources. The numerical method it is found that the planform is down-hexagons for infinite Prandtl numbers and Rayleigh numbers up to at least 15 times the critical value. The motion is also studied for finite Prandtl numbers and small supercritical Rayleigh numbers by using an amplitude expansion. It turns out that a small
subcritical regime exists. It also emerges that for Prandtl numbers less than 0.25 the stable planform is up-hexagons. In §3 a necessary condition in order to obtain a hexagonal planform is derived when the coefficients in the differential equations are a function of the vertical co-ordinate \( z \).

Geoffrey McKay (1979) studied the stability of an internally heated horizontal water layer, with prescribed heat flux on the lower surface and constant upper surface temperature, is studied using a nonlinear energy analysis. The density of the fluid is assumed constant except in the buoyancy term. In this term we will employ an equation of state which assumes cubic dependence on temperature.

G. Z Gershuni, E. M. Zhukhovitskii and A. K. Kolesnikov (1985) have studied the convective motion of a nonisothermal fluid in a gravity field in a vibrating cavity is caused by two mechanisms: the usual static mechanism and a vibrational mechanism. The same mechanisms are also responsible for mechanical equilibrium crisis under the conditions in which such equilibrium is possible. The problems of vibrational-convective stability examined so far relate to cases in which the nonisothermicity was created by specifying the temperature at the boundaries of the region. The present study is concerned with the vibrational-convective stability of a fluid in which the temperature nonuniformity is created by internal heat generation.

C. Israel-Cookey, V. B. omubo-pepple and B. I. Obil (1994) have investigates the condition leading to the onset of stationary convection in a low Prandtl number horizontal fluid layer in a porous medium heated from below with internal heat source. The internal heat source is taken as directly proportional to the temperature leading to sinusoidal temperature gradient in the fluid layer. The effects of heat generation, porosity parameter and different Prandtl numbers, \( Pr \) are presented. The results show that the onset
of stationary instability is hastened by increasing values of the internal heat generation as well as increments in the Prandtl number. Further, increases in the porosity parameter delayed the onset of stationary instability.

A. Khalili and I. S. Shivakumara (1994) have studied the effect of Onset of convection in a horizontal porous layer is investigated including the effects of through-flow and a uniformly distributed internal heat generation for different types of hydrodynamic boundary conditions. The instability parameter is either a Darcy-external or Darcy-internal Rayleigh number which has been determined numerically using Galerkin technique. When the boundary conditions at the top and bottom are not identical, it is found that a small amount of through-flow in one particular direction destabilizes the system in the absence of heat generation. However, in the presence of an internal heat generation, through-flow in one direction destabilizes the system even if the boundary conditions at the two boundaries are of the same type and the destabilization is found to be more with the increase in internal Rayleigh number.

C. Parthiban and Prabhomani R. Patil (1995) studied the effect of horizontal temperature gradients due to non-uniform heating of the boundaries on the onset of convection in a fluid saturating a porous medium with uniformly distributed internal heat sources is studied using Galerkin method. The results reveal that (i) convection sets in via stationary longitudinal modes, (ii) the presence of internal heat sources promotes the onset of convection, its effect being more at higher horizontal gradients, (iii) stationary transverse modes are not possible in a system with internal heat sources, (iv) transverse oscillatory modes are possible only for small
magnitudes of the internal heat source parameter $\eta$, (v) the critical Rayleigh number increases with horizontal gradients.

Ali Nouri-Borujerdi, Amin R. Noghrehabadi D. Andrew and S. Rees (1997) have studied the onset of free convection in a horizontal fluid-saturated porous layer with uniform heat generation. Attention is focused on cases where the fluid and solid phases are not in local thermal equilibrium, and where two energy equations describe the evolution of the temperature of each phase. Standard linearized stability theory is used to determine how the criterion for the onset of convection varies with the inter-phase heat transfer coefficient, $H$, and the porosity-modified thermal conductivity ratio, $\gamma$. We also present asymptotic solutions for small values of $H$. Excellent agreement is obtained between the asymptotic and the numerical results.

C.Parthiban and PrabhamaniR. Patil (1997) studied the Onset of convection in a fluid saturating a horizontal layer of an anisotropic porous medium with internal heat source subjected to inclined temperature gradient is studied using Galerkin technique. The analysis reveals a number of interesting results.

Shivakumara I. S and S. P Suma (2000) have studied the throughflow and internal heat generation effects on the onset of convection in an infinite horizontal fluid layer are investigated. The boundaries are considered to be rigid and perfectly conducting. The resulting eigenvalue problem is solved by using the Galerkin method, and the effects of various parameters in the stability results are analyzed. The results indicate that the stability of the system is significantly affected by both throughflow and internal heat generation in the fluid layer. The Prandtl number comes into play due to the presence of throughflow and it has a profound effect on the stability of the system. They found that, in the presence of internal heating,
throughflow in one direction supresses convection while throughflow in the other direction encourages it.

Sherin M. Alex, Prabhamani R. Patil ,and K. S. Venkatakrishnan (2001) have studied the effect of onset of convection in a horizontal fluid saturated isotropic porous layer, induced by inclined temperature gradient and internal heat source, subject to a gravity field varying linearly with location along the gravitational acceleration direction, is investigated using Galerkin technique. The boundaries are assumed to be impermeable and perfectly conducting. It is seen that the value of the variable gravity parameter plays a decisive role on the onset of convection. When the variable gravity parameter is zero and positive, an increase in the heat generation due to internal heat source advances the onset of convection in both the presence and absence of inclined temperature gradient. On the other hand, when the variable gravity parameter is negative, the opposite effect is seen. Further, it is observed that at the onset of convection the favourable mode is always the stationary longitudinal mode.

N.F. Veitishchev (2004) have studied On the basis of a numerical simulation of convection in a horizontal fluid layer with a uniform heat source it is concluded that the convective heat flux is constant over the entire convection layer not only in the case of steady-state external conditions but also in the case of heating (cooling) of the fluid layer at a constant rate. The convective heat flux is mainly determined by the Rayleigh number and depends only slightly on the layer heating (cooling) rate.

Yuji Tasaka and Yasushi Takeda (2005) have studied the effects of a heat source distribution on natural convection induced by internal heating are studied by using simplified models of the distribution. A linear stability analysis is made to study the effects on critical Rayleigh number and critical
wavenumber. The total amount of heat generation to set convection and the asymmetry in the convective motion are discussed for two extreme cases of heat source distribution. Effect of additional bottom wall heating is also investigated on the critical condition and the asymmetry of the convective motion.

P. G. Siddheshwarand S. Pranesh(2005) have studied the effect of Suction-Injection-Combination (SIC) on the linear stability of Rayleigh-Benard Marangoni convection in a horizontal layer of an Boussinesq fluid with suspended partials confined between an upper free adiabatic boundary and a lower rigid isothermal/adiabatic boundary is considered. The Rayleigh-Ritz technique is used to obtained the eigen value. The influence of various parameters on the onset of convection has analysed. It is found that the effect of Prandtl number on the stability is the system is depend on the SIC being pro-gravity or anti-gravity. A similar Pe-sensitivity is found in respect of the critical wave number. It is observe that the fluid layer with suspended partials heated from below is more stable compared to the classical fluid layer without suspended particles. The problem has possible application in microgravity situation.

Norihan Arifin and Haliza Rosali(2007) they used a numerical technique to analyse the onset of Marangoni convection in a horizontal layer of electrically-conducting fluids heated from below and cooled from above in the presence of a uniform vertical magnetic field. The top surface of a fluid is deformable free and the bottom boundary is rigid and free-slip. The critical values of the Marangoni number for the onset of Marangoni convection are calculated and later it is found

F. I. Dragomirescu and A. Georgescu(2008) two Galerkin method are applied to a problem of convection with uniform internal heat source. With
each methods analytical results are obtained and discussed. They concern the parameters representing the heating rate. Numerical results are also given and they agree well with the existing ones to be critically dependent on the Hartmann, crispation and bond number. They found that the presence of Magnetic field always has a stabilizing effects of increasing the critical Marangoni number when the free surface is non-deformable. The free surface is deformable, then there is a range where the critical Marangoni number will have unstable modes no matter how large magnetic field becomes.

Z. Siri, Z. Mustafa and Hashim. I. (2009) has studied the effect of a feedback control on the onset of oscillatory Benard-Marangoni instability in a rotating horizontal fluid layer is considered theoretically using linear stability theory. It is demonstrated that generally the critical Marangoni number for transition from the no-motion to the motion state can be drastically increased by the combined effects of feedback control and rotation.

Z. Hashim (2009) have studied the effect of a feedback control on the onset of oscillatory Benard-Marangoni instability in a rotating horizontal fluid layer is considered theoretically using linear stability theory. It is demonstrated that generally the critical Marangoni number for transition from the no-motion (conduction) to the motion state can be drastically increased by the combined effects of feedback control and rotation.

Jumpei Takahashi, Yuji Tasaka, Yuichi Mur, Yasushi Takeda and Takatoshi Yanagisawa (2010) have studied the flow structure in a convection cell with an internally heated layer by PIV to elucidate the convection cell transition mechanisms. The vertical velocity component is determined and the cell behaviour with respect to Rayleigh number is investigated quantitatively. Cell expansion process is described as a
consequence of development of the descending flow at the centre of cells. The results suggest that a spoke-like structure is stable in this system in ideal conditions and a double-cell structure is formed when there are restrictions on the system, i.e. finite lateral boundaries.

2.4 Rayleigh-Benard Convection in a Micropolar Fluid

Early pioneering insights concerning the studies on Rayleigh-Benard convection in a micropolar fluid began from the works of Datta and Sastry (1976). They studied thermal instability of a horizontal layer of micropolar fluid heated from below and obtained exact solution. They found that the plot of Rayleigh number versus wave number has two branches of stability. They have also shown that heating from below and above could lead to stationary convective instabilities.

Ahmadi (1976) has studied the stability of a micropolar fluid heated from below by employing a linear theory as well as energy method and showed that the critical Rayleigh number as derived from energy method is identical with the linear limit. He has concluded that the micropolar fluid layer heated from below is more stable as compared with the classical viscous fluid.

Rama Rao (1980) has studied the thermal instability of a thermal conducting micropolar fluid layer under the influence of a transverse magnetic field for rigid surfaces. The problem is solved using finite-difference and Wilkinson iteration techniques. He has found that the instability set in for adverse and favourable temperature gradients.

Lebon and Perez-Garcia (1981) treated the convective instability of a micropolar fluid layer heated from below within the frame work of Serrin-
Joseph’s energy method and displayed subcritical instability in the presence of a coupling between temperature and microrotation. Perez-Garcia et al (1981) have pointed out that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stability (PES) holds. Further, they have also showed that if the linear differential system of balance equations is self–adjoint the PES is automatically fulfilled.

Perez-Garcia and Rubi (1982) have examined the possibility of overstable motions of micropolar fluids heated from below. Their study reveals that the overstable motion will be observed only for fluids with large coupling parameter $\delta$, and if $\delta$ is small the frequency, $\omega$, cannot be positive, so that overstable motions are not possible. They have also pointed out that in the presence of thermal and micropolar effects, the corresponding linear differential system is not self-adjoint and therefore, the PES may not be fulfilled. Consequently, micropolar fluids may present oscillatory motions.

Bhattacharyya and Jena (1983) have investigated the critical study on the stability of a layer of micropolar fluid heated from below with free boundaries. Their analysis shows that the method by which the previous investigators (Datta and Sastry 1976 and Perez-Garcia and Rubi 1982) obtained the critical Rayleigh number have not been justified and the final result obtained thereby seems to be erroneous. They have shown that the overstable motion, which is shown to be possible Perez-Garcia and Rubi 1982) is not justified. Bhattacharyya and Jena (1983) proved the validity of the principle of exchange of stability for this problem.

Bhattacharyya and Jena (1984) have investigated thermal instability of a horizontal layer of micropolar fluid with heat source. They found that heat source and heat sink have the same destabilising effect in micropolar fluid. Further it has been observed that though the vertical component of velocity
and the curl of microrotation do not vanish anywhere between the two boundaries for heat source parameter, $Q = 0$, they vanish even for a small change in the value of $Q$.

Bhattacharyya and Jena (1985) have investigated the convective stability of a horizontal layer of an incompressible micropolar fluid heated from below and rotating about a vertical axis for free-free boundaries after introducing the corrections to the basic equations considered by Sastry and Rao (1983). They have shown that the rotation has a destabilizing effect.

Payne and Straughan (1989) studied the critical Rayleigh numbers for oscillatory and non-linear convection in an isotropic thermomicropolar fluid. They have shown that for a certain sign of the thermal interaction coefficient it is possible to analyse the problem. They also pointed out that the striking feature is that stationary convection is seen to be likely the physically realizable mode whereas oscillatory convection is likely to occur at very high Rayleigh number.

Lindsay and Straughan (1992) have investigated stationary penetrative convection in an incompressible micropolar fluid heated from above and with its lower boundary at a fixed temperature. Linear and non-linear stability results have been obtained for a series of upper surface temperature and values of the micropolar parameter. The energy analysis strongly suggests that micropolar parameter should be small for optimal results.

Franchi and Straughan (1992) have established the non-linear stability for thermal convection in a micropolar fluid with temperature dependent viscosity. They have employed a generalized energy analysis, which requires an intrinsic use of embedding inequalities.

Qin and Kaloni (1992) studied a thermal instability problem in a rotating micropolar fluid heated from below. Their study reveals that the
values of various micropolar parameters and for low values of the Taylor number, the rotation has a stabilizing effect. Also they have pointed out that the micropolar parameters contribute to the condition deciding whether stationary convection or oscillatory convection will prevail.

Ming-I Char, Ko-Ta Chiang and Jong-Jhy Jou (1994) have studied onset of oscillatory instability of Benard-Marangoni convection in a horizontal fluid layer, subject to the Coriolis force and internal generation, is investigated. The upper surface is deformably free and the lower surface is rigid. The characteristic equation of the perturbed state are solved numerically, using the Runge-Kutta-Gill’s shooting method and thr Broyden’s method. The Crispation number C is significant for the occurrence of oscillatory modes. The result show that smaller absolute values of critical Marangoni number $M_c$ and frequency $\sigma_{ic}$ taken place at large value of Crispation number C. The effect of the rotation is stabilizing, while that of the internal generation is strongly destabilizing. The Biot number $B_i$ and the Bond number Bo increases the critical condition. For oscillatory modes, the Prandtl number Pr would decreases the stability.

Tsan-Hui Hsu and Cha’o-Kuang Chen (1996) have investigated the steady, laminar, natural convection flow of a micropolar flow in an enclosure numerically. The transport equations for vorticity, angular momentum and energy are solved with the aid of the cubic spline collocation method.

Hassanien et al(1997) have examined theoretically the steady free convection in micropolar fluids. They have considered four different flows, namely, a vertical isothermal surface, vertical surface with uniform heat flux, a plane plume and a wall plane. The numerical solutions for the
governing equations are presented for a range of values of the material properties and Prandtl number of the fluid.

Ming-I Char, Ko-Ta Chiang (1997) investigated the effect of internal heat generation on the stability of the Benard-Marangoni convection in a horizontal fluid layer with a deformable upper free surface. The stability analysis in this study is based on the linear stability theory. The eigenvalue equations obtained from the analysis are solved by using the fourth-order Runge-Kutta-Gill method with the shooting technique. The results indicate that the stability of Benard-Marangoni convection is significantly affected by internal heat generation in the fluid layer and by surface tension at the upper free surface. There are two different kinds of instability mode: the thermal mode and the surface tensile mode. At lower values of the crispation number $C$, the instability is dominated by the thermal mode. At higher values of $C$, the system becomes more unstable and creates the surface tensile mode, which is induced by the surface tensile effect. The Crispation number $C$ at the transition between the thermal and the surface tensile modes decreases as the value of internal heat generation increases and that of thermal buoyance decreases. The bond number $B_0$ at the mode transition increases due to the existence of the internal heat generation. In addition, the system becomes more stable when the Biot number $Bi$ of the upper free surface increases.

Rees and Pop (1998) have examined theoretically the steady free convection from a vertical isothermal plate immersed in a micropolar fluid. The governing non-similar boundary layer equations are derived and these equations are solved numerically by the Keller-box method.
Y. Narasimha Murty and V. V. Ramana Rao (1998) have studied the effect of throughflow and uniform magnetic field on the onset of Marangoni convection in a horizontal layer of micro-polar fluid bounded below by a rigid isothermal surface and above by a non-deformable free adiabatic surface, for marginal state is studied. The conditions for the onset of instability occurring via stationary convective modes are obtained with the help of Galerkin method. It is observed that both stabilizing and destabilizing factors due to constant vertical throughflow can be enhanced by magnetic field.

Thermal instability in a micropolar fluid has been studied by Siddeshwar and Pranesh (1998a, b). The linear stability analysis is carried out by them and studied the effect of magnetic field on the onset of convection in a Rayleigh – Benard convection and also studied the effect of non-uniform temperature gradients on the onset of convection for different boundary combinations.

Murthy and Yedidi (1999) have investigated the effect of through flow and Coriolis force on convective instabilities in a micropolar fluid layer heated from below for free-free isothermal and micro-rotation free boundaries. By employing a lower order Galerkin approximation they have obtained the eigenvalue for the stationary case and observed that both stabilising and destabilising factors due to constant vertical through flow can be enhanced by rotation.

Free convective flow of thermomicropolar fluid along a vertical plate with non-uniform surface temperature and surface heat flux has been studied by Hassanien et al (1999). They have determined the effects of microintertia density and the vortex viscosity on laminar free convection boundary layer flow by numerical methods.
Siddeshwar and Pranesh (1999, 2000) have studied the effect of temperature and gravity modulation on the onset of Rayleigh – Benard convection in a fluid with internal angular momentum.

P. G. Siddheswar and S. Pranesh (2000) have studied the effect of Suction – Injection – Combination (SIC) on the linear stability of Rayleigh – Benard Marangoni convection in a horizontal layer of a Boussinesq fluid with suspended particles confined between an upper free / adiabatic boundary and a lower rigid isothermal / adiabatic boundary is considered. The Rayleigh – Ritz technique is used to obtain the eigenvalues. The influence of various parameters on the onset of convection has been analysed. It is found that the effect of Prandtl number on the stability of the system is dependent on the SIC being pro – gravity or anti – gravity. A similar Pe – sensitivity is found in respect of the critical wave number. It observed that the fluid layer with suspended particles heated from below is more stable compared to the classical fluid layer without suspended particles. The problem has possible applications in microgravity situations.

P. G. Siddheswar and S. Pranesh (2000) have studied the effect of a non – uniform temperature gradient and magnetic field on the onset of convection driven by surface tension in a horizontal layer of Boussinesq fluid with suspended particles confined between an upper free / adiabatic boundary and a lower rigid / isothermal boundary have been considered. A linear stability analysis is performed. The microrotation is assumed to vanish at the boundaries. The Galerkin technique is used to obtain the eigenvalues. The influence of various parameters on the onset of convection has been analysed. Six different non – uniform temperature profiles are considered and their comparative influence on onset is discussed. It is observed that the electrically conducting fluid layer with suspended particles heated from
below is more stable compared to the classical electrically conducting fluid without suspended particles. The critical wave number is found to be insensitive to the changes in the parameters but sensitive to the changes in the Chandrasekhar number. The problem has possible applications in microgravity space situations.

Siddheshwar and Srikrisna (2000) have studied the suction-injection-combination on the onset of Rayleigh-Benard convection in suspensions using the Rayleigh-Ritz method. The eigenvalues have been obtained for different boundary combinations. They found that the effect of SIC on the critical eigenvalue is shown to be dependent on whether it is gravity-aligned or anti-gravity.

P. G Siddheswar and S. Pranesh (2001) have studied the role of magnetic field in the inhabitation of natural convection driven by combined buoyancy and surface tension forces in a horizontal layer of Boussinesq fluid with suspended particles confined between an upper free / adiabatic boundary and a lower rigid / isothermal boundary have been considered in $1g$ and $\mu g$ situations. The inhibition of convection is caused by a stationary and uniform magnetic field parallel to the gravity field. The magnetically – inert suspended particles are not directly by the magnetically responding carrier fluid in which they are suspended. A linear stability analysis is performed. The Rayleigh – Ritz technique is used to obtain the eigenvalues. The influence of various parameters on the onset of convection has been analysed. Six different reference steady – state temperature profiles are considered and their comparative influence on onset is discussed. Treating Marangoni number as the critical parameter it is shown that any particular infinitesimal disturbance can be stabilized with a sufficiently strong magnetic field. It is observed that the electrically conducting fluid layer with
suspended particles heated from below is more stable compared to the classical electrically conducting fluid without suspended particles. The critical wave number is found to be insensitive to the changes in the suspension parameters but sensitive to the changes in the Chandrasekhar number. The problem has possible space applications.

Y. N. Murty (2001) have studied the effect of throughflow and magnetic field on the onset of Bénard convection in a horizontal layer of micropolar fluid permeated between two rigid, isothermal and micro-rotation free boundaries is studied. The determination of the critical Rayleigh number entails in solving the eigenvalue problem numerically for which the single-term Galerkin method is employed. It is established that both stabilizing and destabilizing factors can be enhanced by throughflow.

S. Pranesh (2003) have studied the effect of Suction – Injection – Combination (SIC) on the linear stability of Rayleigh – Benard convection in a horizontal layer of a Boussinesq micropolar fluid is studied using Rayleigh – Ritz technique. The eigenvalues are obtained free – free, rigid – free and rigid – rigid velocity boundary combinations with isothermal and adiabatic temperature conditions on the spin – vanishing boundaries. The eigenvalues are also obtained for lower rigid isothermal and upper free adiabatic boundaries with vanishing spin. The influence of various micropolar fluid parameters on the onset of convection has been analysed. It is found that the effect of Prandtl number on the stability of the system is dependent on the SIC being pro – gravity or anti – gravity. A similar Pe – sensitivity is found in respect of the critical wave number. It observed that the micropolar fluid layer heated from below is more stable compared to the classical fluid layer.
D. Srinivasacharya and M. Shiferaw (2004) have investigated MHD flow of micropolar fluid in a rectangular duct with hall and ion slip effects. The steady flow of incompressible and electrically conducting micropolar fluid flow through a rectangular channel is considered taking Hall and ionic effects into consideration. An external uniform magnetic field is applied which is directed arbitrary in a plane perpendicular to the flow direction. The governing partial differential equations are solved numerically using finite difference method, and the effects of micropolar parameters, magnetic parameter, Hall parameter and ion slip parameter on the velocity and microrotation are discussed.

R. Nazar and N. Amin (2005) Numerical solution for the steady laminar free convection boundary layer flow over a horizontal circular cylinder subjected to a constant surface heat flux in a micropolar fluids are presented. The governing boundary layer equations are first transformed into a non-dimensional form. These equation are then transformed into a set of nonsimilar boundary layers, which are solved numerically using a very efficient implicit finite-difference method known as the Keller-box scheme. The obtained solution for the material parameter K=0 and different

C. I. Christov and P. M. Jordan (2005) have investigated revisit the Maxwell-Cattaneo law of finite-speed heat conduction. We point out that the usual form of this law, which involves a partial time derivative, leads to a paradoxical result if the body is in motion. We then show that by using the material derivative of the thermal flux, in lieu of the local one, the paradox is completely resolved. Specifically, that using the material derivative yields a constitutive relation that is Galilean invariant. Finally, we show that under this invariant reformulation, the system of governing equations, while still
hyperbolic, cannot be reduced to a single transport equation in the multidimensional case.

P. G Siddheswar and S. Pranesh (2005) have studied the linear and weakly non–linear analyses of Rayleigh – Benard convection in a micropolar fluid are made. The condition for stationary and oscillatory modes in the case of linear theory is obtained using a Rayleigh – Ritz technique for general boundary conditions on velocity and temperature. The non–linear analysis for the case free–free isothermal boundaries is based on the truncated representation of Fourier series. A striking feature of the study is that stationary convection is shown to be physically realizable mode whereas oscillatory convection is theoretically predicated to occur for certain values of the parameters. The Nusselt number is calculated for different values of Rayleigh number and other parameters arising in the problem. There is no indication of subcritical motions from the non linear analysis.

Anuar Ishak, Roslinda Nazar and Ioan Pop (2006) have studied the effect of buoyancy forces on fluid flow and heat transfer over a horizontal plate in a steady, laminar and incompressible micropolar fluid has been investigated. The wall temperature is assumed to be inversely proportional to the square root of the distance from the leading edge. The set of similarity equations has been solved numerically using the Keller-box method, and the solution is given for some values of buoyancy parameter, material (micropolar) parameter and Prandtl number. It is found that dual solutions exist up to certain negative values of buoyancy parameter (decelerated flow) for all values of micropolar parameter and Prandtl number considered in this study. Beyond these values, the solution does no longer exist. Moreover, it is found that there is no local heat transfer at the wall except in the singular
point at the leading edge, although the wall temperature is different from the free stream temperature.

Mohd Nasir Mahmud, Ruwaidiah Idris, and Ishak Hashim (2006) have investigated the effect of a magnetic field on the onset of Marangoni convection in a micropolar fluid with the presence of a uniform vertical magnetic field and suspended particles; thermocapillary instability in a horizontal liquid layer is investigated. The resulting eigenvalues is solved by the Galerkin technique for various basic temperature gradients. They found that the presence of magnetic field always has a stability effect of increasing the critical Marangoni number.

S. Pranesh (2008) have studied the effect resulting from the substitution of the classical Fourier law by the non-classical Maxwell – Cattaneo law in Rayleigh – Benard convection in micropolar fluid is studies. Analytical solution is obtained and found the classical approach predicts an infinite speed for the propagation of heat. The present non-classical theory involves a wave type heat transport (SECOND SOUND) and does not suffer from the physically unacceptable drawback of infinite heat propagation speed.

Haliza Othman, seripah Awang Kechil and Ishak Hashim (2008) The effect of feedback control on the onset of steady surface-tension-driven (Marangoni) convection in a horizontal fluid layer in the presence of heat generation is considered theoretically using stability theory. The fluid layer, heated from below, is bounded above by a flat free upper surface and below by a rigid plane boundary. The role of the controller gain parameter on the stability of the fluid layer is investigated.
Nor fah Bachok, Norihan and Fadzilah (2008) have studied the effect of control on the onset of Marangoni-Benard convection in a horizontal layer of fluid with internal heat generation from below and cooled from above is investigated. The resulting eigenvalue problem is solved exactly. The effect of control are studied by examining the critical Marangoni number and wave numbers. Ti is found that the onset of Marangoni-Benard convection with internal heat generation can be delayed through the use of control.

S. Pranesh (2008) have studied the effects resulting from the substitution of the classical Fourier law by the non-classical Maxwell – Cattaneo law in Rayleigh – Benard convection in second order fluid is studies. Coleman – Noll constitutive equation is used to give a viscoelastic correction. The eigenvalue is obtained for free – free isothermal boundary combination. The classical approach predicts an infinite speed for the propagation of heat. The present non-classical theory involves a wave type heat transport (SECOND SOUND) and does not suffer from the physically unacceptable drawback of infinite heat propagation speed.

K. S. Mekheimer (2008) has studied the effect of the induced magnetic field on peristaltic transport of an incompressible conducting micropolar fluid in a symmetric channel. The flow analysis has been developed for low Reynolds number and long wavelength approximation. Exact solutions have been established for the axial velocity, microrotation component, stream function, magnetic-force function, axial-induced magnetic field, and current distribution across the channel. Expressions for the shear stresses are also obtained. The effects of pertinent parameters on the pressure rise per wavelength are investigated by means of numerical integrations; also we study the effect of these parameters on the axial pressure gradient, axial-induced magnetic field, as well as current
distribution across the channel and the nonsymmetric shear stresses. The phenomena of trapping and magnetic-force lines are further discussed.

D. Srinivasacharya and M. Shiferaw (2009) have studied Steady flow of an incompressible and electrically conducting micropolar fluid through a diverging channel is studied. The flow is subjected to a uniform magnetic field perpendicular to the flow direction. Perturbation solutions have been obtained for the velocity and microrotation components in terms of effective Reynolds number. The profiles of velocity and microrotation components are presented for different micropolar fluid parameters and magnetic parameter. The influence of magnetic parameter on the pressure gradient is also studied.

R. Idris, H. Othman and I. Hashim (2009) have studied Linear stability analysis is performed to study the effect of non-uniform basic temperature gradients on the onset of Benard–Marangoni convection in a micropolar fluid.
CHAPTER III
BASIC EQUATIONS, BOUNDARY CONDITIONS, APPROXIMATIONS, SCALING AND DIMENSIONLESS PARAMETERS

In this chapter, we discuss the relevant basic equations, boundary conditions, approximations, scaling and dimensionless parameters arising in the problems under investigation, in a general manner.

3.1 EQUATIONS OF MOTION FOR MICROPOLAR FLUID:

The theory of micro-fluid introduced by Eringen (1964) deals with a class of fluids, which exhibits certain microscopic effects arising from the local structure and micro-motions of the fluid elements. These fluids can support stress moments and body moments and are influenced by the spin inertia. Consequently new principles must be added to the basic principle of continuous media which deals with

(i) Conservation of micro inertia moments, and

(ii) Balance of first stress moments.

The theory of micro fluids naturally give rise to the concept of inertial spin, body moments, micro-stress averages and stress moments which have no counterpart in the classical fluid theories. In these fluids, stresses and stress moments are functions of deformation rate tensor and various micro-deformation rate tensors.

Eringen (1966) introduced the theory of micropolar fluids, a subclass of microfluids, which exhibit the micro-rotational effects and micro-
rotational inertia. Further, he showed that this class of fluids possesses certain simplicity and elegance in their mathematical formulation, which should appeal to mathematicians. Eringen (1966) derived the equation of motion, constitutive equations and boundary conditions for micropolar fluids. Eringen (1972) introduced the theory of thermomicro fluids by extending his earlier work to include the heat conduction and heat dissipation effects. He also gave the exact non-linear theory. Eringen (1980) introduced a theory of anisotropic fluids based on an extension of micropolar fluid mechanics. Since the micropolar fluid consists of rigid particles of arbitrary shapes and inertia, by using microinertia tensor as the orientation descriptor it is possible to develop a theory of anisotropic fluids. To qualify anisotropic fluids as a suspension theory, Eringen (1991) introduced continuum theory of dense rigid suspensions. The balance laws and constitutive equations of micropolar continuum theory are modified and extended for the characterization of dense rigid suspensions.

The basic equations for the micropolar fluids are

**Continuity equation:**
\[ \nabla \cdot \vec{\dot{q}} = 0, \quad (3.1) \]

**Conservation of linear momentum:**
\[ \rho_o \left[ \frac{\partial \vec{\dot{q}}}{\partial t} + (\vec{\dot{q}} \cdot \nabla) \vec{\dot{q}} \right] = -\nabla P - \rho g \vec{k} + (2\zeta + \eta) \nabla^2 \vec{\dot{q}} + \zeta \nabla \times \vec{\omega} + \mu_m (\vec{H} \cdot \nabla) \vec{H}, \quad (3.2) \]

**Conservation of angular momentum:**
\[ \rho_o I \left[ \frac{\partial \vec{\omega}}{\partial t} + (\vec{\omega} \cdot \nabla) \vec{\omega} \right] = (\lambda' + \eta') \nabla (\nabla \cdot \vec{\omega}) + (\eta' \nabla^2 \vec{\omega}) + \zeta (\nabla \times \vec{\omega} - 2\vec{\omega}), \quad (3.3) \]
Conservation of energy:
\[
\frac{\partial T}{\partial t} + \left(\ddot{q} - \frac{\beta}{\rho_o C_v} \nabla \times \ddot{\omega}\right) \nabla T = \chi \nabla^2 T + Q,
\]  
(3.4)

In addition to the above equations we require the magnetic induction equation:

Magnetic induction equation:
\[
\frac{\partial \tilde{H}}{\partial t} + (\tilde{q} \cdot \nabla) \tilde{H} = (\tilde{H} \cdot \nabla) \tilde{q} + \gamma_m \nabla^2 \tilde{H},
\]  
(3.5)

Equation of state:
\[
\rho = \rho_o [1 - \alpha (T - T_o)].
\]  
(3.6)

Where:

- $\tilde{q}$ is the velocity, $\ddot{\omega}$ is the spin, $T$ is the temperature, $p$ is the hydrodynamic pressure, \(\tilde{H}\) is the magnetic field, $\rho$ is the density, $\rho_o$ is the density of the fluid at reference temperature $T = T_o$, $g$ is the acceleration due to gravity, $\zeta$ is the coupling viscosity co–efficient or vortex viscosity, $\eta$ is the shear kinematic viscosity co–efficient, $I$ is the moment of inertia, $\lambda'$ and $\eta'$ are the bulk and shear spin viscosity co–efficients, $\beta$ is the micropolar heat conduction co–efficient, $C_v$ is the specific heat, $\kappa$ is the thermal conductivity, $\alpha$ is the co–efficient of thermal expansion, $\gamma_m = \frac{1}{\mu_m \sigma_m}$ is the magnetic viscosity ($\sigma_m$ : electrical conductivity and $\mu_m$ : magnetic permeability), $\ddot{\omega}_l = \frac{1}{2} \nabla \times \ddot{q}$, $\ddot{Q}$ is the heat flux vector and $\tau$ is the constant relation time.
3.2 APPROXIMATIONS:

1. The fluid is assumed to be homogeneous and incompressible continuum.

2. When the fluid is at equilibrium, the flow field is isothermal, i.e. the temperature of the fluids is everywhere below the boiling point heading to an equation of state where the density of the fluid is a linear function of temperature according to $\rho = \rho_o[1 - \alpha(T - T_o)]$.

3. The Boussinesq approximation is assumed to be valid. This assumption allows the fluid density to vary only in the buoyancy force term in the momentum equation and elsewhere it is treated as a constant. This is valid provided the velocity of the fluid is much less than that of sound. The basic idea of this approximation is to filter out high frequency phenomena such as sound waves. Since they do not play an important role in transport process.

4. There is no radiation.

5. The gravity acts vertically downwards.

6. The fluid parameters namely viscosity, magnetic viscosity, thermal diffusivity, coupling viscosity, shear kinematic viscosity, micropolar heat conduction parameter and bulk and shear spin velocity are all assumed to be constant.

7. The usual MHD approximations are taken into account.

8. The suspended particles are magnetically non-responding whereas the carrier fluids are magnetically responding.
3.3. BOUNDARY CONDITIONS:

The above equations are to be solved subject to certain containment conditions. In this thesis we have used the following boundary conditions:

3.3.1 Boundary Condition On Velocity:

The boundary conditions on velocity are obtained from mass balance, the no-slip condition and the stress principle of Cauchy, depending on the nature of boundary surfaces of the fluid (rigid or free). The boundary surfaces considered in this thesis are:

(i) Lower surface is rigid and upper surface is free and

In the case of rigid boundaries the boundary conditions on velocity are

\[ w = \frac{\partial w}{\partial z} = 0. \]  (3.7)

3.3.2 Boundary Condition On Temperature:

The boundary condition on temperature will depend on the nature of the boundaries in terms of heat conductivity.

(a) Fixed Surface Temperature:

In this case, we use

\[ T = 0 \]  (3.8)

at the boundaries. This boundary condition is known as isothermal or boundary condition of the first kind.

(b) Fixed Surface Heat Flux:

In this case, we use

\[ \frac{\partial T}{\partial z} = 0 \]  (3.9)
at the boundaries. These boundary conditions are known as adiabatic boundary condition or boundary condition of second kind.

### 3.3.3 Boundary condition on Micro-rotation:

In the case of micropolar fluids in addition to the boundary conditions on velocity and temperature, we have to consider the boundary condition on micro-rotation. The boundary condition on the micro-rotation is spin vanishing boundary condition i.e., we assume that the micro-rotation is zero at the boundaries.

### 3.3.4 Electro-Magnetic Boundary Conditions:

Electromagnetic boundary conditions will also depend on the material of the bounding surface. If the boundary surface is electrically conducting, the tangential component of electric field and normal component of magnetic induction are continuous across the surface, whereas the tangential component of magnetic field is discontinuous in the presence of surface current. In the absence of surface current, the tangential component of magnetic field is also continuous.

\[ H_x = H_y = \frac{\partial H_z}{\partial z} = 0, \]  

(3.10)

at the boundaries.

### 3.4 Scales Used For Non-Dimensionalization:

To know the relative importance of each term in the equation, we make the equations dimensionless. There are two ways of making the equation dimensionless, one is by introducing the characteristic quantities and another is by comparing similar terms. We follow the method of introducing
characteristic quantities. The scales used in the thesis for non-dimensionlization are tabulated below:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Characteristic quantity used for scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>$d^2/\chi$</td>
</tr>
<tr>
<td>Length</td>
<td>$d$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$\chi/d$</td>
</tr>
<tr>
<td>Microrotation</td>
<td>$\chi/d^2$</td>
</tr>
<tr>
<td>Temperature</td>
<td>$\Delta T$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_0$</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>$H_0$</td>
</tr>
<tr>
<td>Pressure</td>
<td>$\rho_o \chi^2/d^2$</td>
</tr>
</tbody>
</table>

3.5 Dimensionless Parameters:

The following dimensionless parameters appear in the thesis:

(i) Rayleigh Number $R$:

The Rayleigh number is defined as

$$R = \frac{\alpha g \Delta T d^3 \rho_o}{(\zeta + \eta)\kappa}.$$  

Where: $\Delta T$ is the temperature difference between two bounding planes, $d$ the thickness of the fluid layer. Physically, the Rayleigh number represents
the measure of buoyancy force to the dissipation force of viscous and thermal dissipation.

(ii) **Coupling Parameter** $N_1$:

Coupling parameter is defined as

$$N_1 = \frac{\zeta}{\zeta + \eta} \quad (0 \leq N_1 \leq 1).$$

Physically, $N_1$ represents the concentration of the suspended particles in the fluid.

(iii) **Couple Stress Parameter** $N_3$:

The couple stress parameter is defined as

$$N_3 = \frac{\eta'}{(\zeta + \eta)d^2} \quad (0 \leq N_3 \leq m),$$

where $m$ is finite, positive real number.

(iv) **Micropolar Heat Conduction Parameter** $N_5$:

The micropolar heat conduction parameter is defined as

$$N_5 = \frac{\beta}{\rho_o C_v d^2} \quad (0 \leq N_5 \leq n),$$

where $n$ is the finite, positive real number.

(v) **Prandtl number** $P_r$:

The Prandtl number is defined as

$$P_r = \frac{\eta + \zeta}{\rho_o K}$$

(vi) **Magnetic Prandtl number** $P_m$:
The Magnetic Prandtl number is defined as

\[ P_m = \frac{\eta + \zeta}{\rho_o \gamma m} \]

(vii) Chandrasekhar number \( Q \):

The Chandrasekhar number is defined as

\[ Q = \frac{\mu_m \bar{H}_o d^2}{\mu_m (\zeta + \eta)} \]

(viii) Internal Rayleigh number \( R_I \):

The Internal Rayleigh number is defined as

\[ R_I = \frac{Q d^2}{\chi \Delta T} \]
CHAPTER – IV
EFFECT OF INTERNAL HEAT GENERATION ON THE ONSET OF RAYLEIGH-BÉNARD MARANGONI MAGNETO-CONVECTION IN A HORIZONTAL LAYER OF FLUID WITH SUSPENDED PARTICLES

The problem of onset of convection driven by the combined effect of buoyancy and surface tension is important. Hence we study the effects of magnetic field on the onset of Rayleigh-Bénard-Marangoni convection in a micropolar fluid Law.

4.1 Mathematical Formulation:

Consider an infinite horizontal layer of a Boussinesquian, electrically conducting fluid, with non-magnetic suspended particle, of depth ‘d’ permeated by an externally applied uniform magnetic field normal to the layer (see figure (1)). A cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards. Let \( \Delta T \) be the temperature difference between the upper and lower boundaries. The body forces acting on the fluid are buoyancy and magnetic field.

The governing equations for the Rayleigh-Bénard situation in a Boussinesquian fluid with suspended particles are

**Continuity equation:**

\[
\nabla \cdot \tilde{q} = 0, \quad (4.1)
\]

**Conservation of linear momentum:**

\[
\rho_o \left[ \frac{\partial \tilde{q}}{\partial t} + (\tilde{q} \cdot \nabla) \tilde{q} \right] = -\nabla P - \rho g \hat{k} + (2\zeta + \eta)\nabla^2 \tilde{q} + \zeta \nabla \times \tilde{\omega} + \mu_m (\vec{H} \cdot \nabla) \vec{H}, \quad (4.2)
\]
Conservation of angular momentum:
\[
\rho_o I \left[ \frac{\partial \vec{\omega}}{\partial t} + (\vec{q} \cdot \nabla) \vec{\omega} \right] = (\lambda' + \eta') \nabla (\nabla \cdot \vec{\omega}) + (\eta' \nabla^2 \vec{\omega}) + \zeta (\nabla \times \vec{q} - 2 \vec{\omega}), \tag{4.3}
\]

Conservation of energy:
\[
\frac{\partial T}{\partial t} + \left( \vec{q} - \frac{\beta}{\rho_o C_v} \nabla \times \vec{\omega} \right) \cdot \nabla T = \chi \nabla^2 T + Q, \tag{4.4}
\]

Magnetic induction equation:
\[
\frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} = (\vec{H} \cdot \nabla) \vec{q} + \gamma_m \nabla^2 \vec{H}, \tag{4.5}
\]

Equation of state:
\[
\rho = \rho_o [1 - \alpha (T - T_o)]. \tag{4.6}
\]

The aim of this chapter is to investigate the stability of a quiescent state to infinitesimal perturbations superposed on the basic state. The basic state of the fluid being quiescent is described by
\[
q_0 = (0,0,0), \quad \vec{w}_b = (0,0,0), \quad \vec{H} = \vec{H}_0 k, \quad P = P_b(z), \quad \rho = \rho_b(Z) \quad \left\{ \begin{array}{l}
T = T_b(z).
\end{array} \right. \tag{4.7}
\]

More details on this quasi-static approximation in equation (4.8) can be found in the references of Lebon and Cloot (1981). Equations (4.2), (4.4) and (4.6) in the basic state specified by equation (4.7) respectively become
\[ \begin{align*}
\frac{dP_b}{dz} &= -\rho_b g \hat{k}, \\
\frac{d^2 T_b}{dZ^2} &= -\frac{Q}{\chi}, \\
\rho_b &= \rho_o [1 - \alpha (T_b - T_o)]
\end{align*} \]

Equations (4.1), (4.3) and (4.5) are identically satisfied by the concerned basic state variables.

Solving equation (4.4) for \( T_b \) after basic equation state using the Boundary condition

\[ T_b = T_1 \text{ at } Z = 0 \]
\[ T_b = T_0 \text{ at } Z = d \]

\[ T_b = \frac{-Q Z^2}{2 \chi} + \left( \frac{Q_1 d}{2 \chi} - \frac{\Delta T}{d} \right) Z + T_0 \]

Equation (4.9)

We now superpose infinitesimal perturbations on the quiescent basic state and study the instability.
4.2 Linear Stability Analysis:

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

\[
\begin{align*}
\bar{q} &= \bar{q}_b + \bar{q}' , \quad \bar{\omega} = \bar{\omega}_b + \bar{\omega}' , \quad P = P_b + P', \\
\rho &= \rho_b + \rho' , \quad T = T_b + T', \quad \bar{H} = \bar{H}_b + \bar{H}'.
\end{align*}
\] (4.10)

The primes indicate that the quantities are infinitesimal perturbations and subscript ‘b’ indicates basic state value.

Substituting equation (4.10) into equations (4.1) – (4.6) and using the basic state (4.8) and (4.9), we get linearised equation governing the infinitesimal perturbations in the form:

\[
\nabla \cdot \bar{q}' = 0, \quad (4.11)
\]

\[
\rho_0 \left[ \frac{\partial \bar{q}'}{\partial t} \right] = -\nabla P' - \rho' g \hat{\mathbf{k}} + (2\zeta + \eta)\nabla^2 \bar{q}' + (\zeta \nabla \times \bar{\omega}') + \mu_m H_0 \frac{\partial \bar{z}'}{\partial z} \hat{\mathbf{z}'}, \quad (4.12)
\]

\[
\rho_0 \left[ \frac{\partial \bar{\omega}'}{\partial t} \right] = (\lambda' + \eta')\nabla \left( \nabla \bar{\omega}' \right) + (\eta' \nabla^2 \bar{\omega}') + \zeta (\nabla \times \bar{q}' - 2\bar{\omega}'), \quad (4.13)
\]

\[
\frac{\partial T'}{\partial t} = \frac{dT_b}{dz} \left[ \bar{q}' - \frac{\beta}{\rho_0 C_V} \nabla \times \bar{\omega}' \right] - \chi \nabla^2 T', \quad (4.14)
\]
\[
\frac{\partial \tilde{H}'}{\partial t} = \left( H_o \frac{\partial W}{\partial z} \right) + \gamma_m \nabla^2 \tilde{H}', \quad (4.15)
\]

\[
\rho' = -\alpha \rho_o T'. \quad (4.16)
\]

The perturbation equation (4.12), (4.13), (4.14) and (4.15) are non-dimensionalised using the following definition:

\[
\begin{align*}
(x^*, y^*, z^*) &= \frac{(x, y, z)}{d}, \quad \omega^* = \frac{\omega'}{\left( \frac{\chi}{d^2} \right)}, \quad P^* = \frac{P'}{P_o} \\
t^* &= \frac{t}{\left( \frac{d^2}{\chi} \right)}, \quad T^* = \frac{T'}{\Delta T}, \quad \tilde{H'}^* = \frac{\tilde{H}'}{H_o}, \quad \Omega^*_z = \frac{\Omega_z}{\left( \frac{\chi}{d^3} \right)} \\
\tilde{q}^* &= \frac{\tilde{q}'}{\chi/d}, \quad \rho^* = \frac{\rho'}{\rho_o}
\end{align*}
\]  
\quad (4.17)

Using equation (4.16) in (4.12), Operating curl twice on the resulting equation operating curl once on equation (4.13) and non-dimensionalising the two resulting equation and also equation (4.14).

\[
\frac{1}{P_r} \frac{\partial}{\partial Z} \left( \nabla^2 W \right) = R\nabla_1^2 T + (1 + N_1) \nabla^4 W + N_1 \nabla^2 \Omega_z + \mu_m H_o \frac{\partial H'}{\partial Z} \hat{K} \quad (4.18)
\]

\[
\frac{1}{P_r} \frac{\partial}{\partial Z} (\Omega Z) = N_3 \nabla^2 \Omega_z - N_1 \nabla^2 W - 2N_1 \Omega_z \quad (4.19)
\]
\[
\frac{\partial T}{\partial Z} = \nabla^2 T + W - N_5 \Omega Z - R_I \left( \frac{1}{2} - Z \right) W + R_I N_5 \left( \frac{1}{2} - Z \right) \Omega Z \quad (4.20)
\]

\[
\frac{\partial H'}{\partial Z} = \frac{\partial W}{\partial Z} + \frac{P_r}{P_m} \nabla^2 H Z \quad (4.21)
\]

Where the asterisks have been dropped for simplicity and the non-dimensional parameters \( N_1, N_3, N_5, R, Q, P_r \) and \( C \) are as defined as

\[
N_1 = \frac{\zeta}{\zeta + \eta} \quad \text{(Coupling Parameter)},
\]

\[
N_3 = \frac{\eta'}{(\zeta + \eta)d^2} \quad \text{(Couple Stress Parameter)},
\]

\[
N_5 = \frac{\beta}{\rho_o \gamma_v d^2} \quad \text{(Micropolar Heat Conduction Parameter)},
\]

\[
P_r = \frac{\zeta + \eta}{\rho_o \kappa} \quad \text{(Prandtl Number)},
\]

\[
P_m = \frac{(\eta + \zeta)}{(\rho_o \gamma_m)} \quad \text{(Magnetic Prandtl number)},
\]

\[
R = \frac{\rho_o \alpha g \Delta T d^3}{\kappa (\zeta + \eta)} \quad \text{(Rayleigh number)},
\]

\[
Q = \frac{\mu_m H_o d^2}{\gamma_m (\zeta + \eta)} \quad \text{(Chandrasekhar number)},
\]

\[
R_I = \frac{Q d^2}{\chi \Delta T} \quad \text{(Internal Rayleigh Number)}
\]
The infinitesimal perturbation $W, \Omega_z$ and $T$ are assumed to be periodic waves (see Chandrasekhar 1961) and hence these permit a normal mode solution in the form

$$\begin{bmatrix} W \\ \Omega_z \\ T \end{bmatrix} = \begin{bmatrix} W(z) e^{i(lx + my)} \\ G(z) e^{i(lx + my)} \\ T(z) e^{i(lx + my)} \end{bmatrix}$$

(4.22)

Where $l$ and $m$ are horizontal components of the wave number $\tilde{a}$.

Substituting equation (4.22) into equation (4.18) – (4.21), after eliminating $Z_H$ between equation (4.18) and equation (4.21), we get

$$-Ra^2T + (1 + N_1)(D^2 - a^2)^2 + N_1(D^2 - a^2)G - QD^2W = 0,$$  

(4.23)

$$N_3(D^2 - a^2)G - N_1(D^2 - a^2)W - 2N_1G = 0$$

(4.24)

$$ (D^2 - a^2)T + W - N_5G - R_I\left(\frac{1}{2} - Z\right)W + R_I N_5\left(\frac{1}{2} - Z\right)G = 0$$

(4.25)

Where

$$D = \frac{d}{dz}$$

The sets of ordinary differential equations (4.23)-(4.25) are approximations based on physical considerations to the system of partial differential equations (4.18)-(4.20). Although the relationship between the solutions of the governing partial differential equations and the corresponding ordinary differential equations has not been established, these
linear models reproduce qualitatively the convective phenomena observable through the full system.

### 4.3 Application of Galerkin Method:

In this section we apply the Galerkin method to equation (4.23), (4.24) and (4.25) that gives general results on the eigen value of the problem for various basic temperature gradients using simple, polynomial, trial functions for the lowest eigen value. We obtain an approximate solution of the differential equations with the given boundary conditions by choosing trial functions for velocity, microrotation and temperature perturbations that may satisfy some of the boundary conditions but may not exactly satisfy the differential equations. This leads to residuals when the trial functions are substituted into the differential equations. The Galerkin method requires the residual to be orthogonal to each individual trial function.

In the Galerkin procedure, we expand the velocity, microrotation and temperature by,

\[
\begin{align*}
W(z,t) &= \sum A_i(t)W_i(z) \\
G(z,t) &= \sum B_i(t)G_i(z) \\
T(z,t) &= \sum E_i(t)T_i(z)
\end{align*}
\]

(4.26)

where:

\( W_i(z), G_i(z) \) and \( T_i(z) \) are polynomials in \( z \) that generally have to satisfy the given boundary conditions.

For the single term Galerkin expansion technique we take \( i = j = 1 \).
Multiplying equation (4.23) by $W$, equation (4.24) by $G$ and equation (4.25) by $T$, integrating the resulting equation by parts with respect to $z$ from 0 to 1 and taking $W = AW_1$, $G = BG_1$ and $T = ET_1$ in which $A$, $B$ and $E$ are constants with $W_1$, $G_1$ and $T_1$ are trial functions. This procedure yields the following equation for the Rayleigh number $R$:

$$R = \frac{M_1 [(1 + N_1)M_2 X_1 + N_1^2 M_3 M_4 - Q M_5 X_1]}{a^2 * [N_5 N_1 X_2 - M_8 X_1 + R I [X_1 M_{10} - N_5 N_1 M_3 M_{11}]] M_8} \quad (4.27)$$

where

$$M_1 = \langle T_1 (D^2 - a^2) T_1 \rangle$$

$$M_2 = \langle W_1 (D^2 - a^2)^2 W_1 \rangle$$

$$M_3 = \langle G_1 (D^2 - a^2) W_1 \rangle$$

$$M_4 = \langle W_1 (D^2 - a^2) G_1 \rangle$$

$$M_5 = \langle W_1 D^2 W_1 \rangle$$

$$M_6 = \langle G_1 (D^2 - a^2) G_1 \rangle$$

$$M_7 = \langle G_1^2 \rangle$$

$$M_8 = \langle W_1 T_1 \rangle$$

$$M_9 = \langle G_1 T_1 \rangle$$

$$M_{10} = \left\langle \left( \frac{1}{2} - Z \right) W_1 T_1 \right\rangle$$
\[ M_{11} = \left\langle \left[ \frac{1 - Z}{2} \right] G_1 T_1 \right\rangle \]

\[ X_1 = N_3 M_6 - 2 N_1 M_7 \]
\[ X_2 = M_3 M_9 \]

In the equation (4.27), \( \langle - - \rangle \) denotes integration with respect to \( z \) between \( z = 0 \) and \( z = 1 \). We note here that \( R \) is equation (4.27) is a functional and the Euler – Lagrange equations for the extremisation of \( R \) are equations (4.23), (4.24) and (4.25).

If \( R_f = 0 \) in the equation (4.27), we get the expression obtained by Siddheshwar and Pranesh in (1998) as a limiting case.

The value of critical Rayleigh number depends on the boundaries. In this chapter we consider various boundary combinations and these are discussed below.

### 4.4 CRITICAL RAYLIEGH NUMBER FOR ISOTHERMAL BOUNDARIES

We consider three cases:

i. Both boundaries are free (Free – Free)

ii. Both boundaries are rigid (Rigid – Rigid) and

iii. Lower boundary rigid and upper boundary free (Rigid – Free).

In all the above three cases, we take spin – vanishing boundary condition on microrotation.
4.4.1 Critical Rayleigh number when lower boundary is rigid and upper boundary is free:

The boundary conditions are

\[ \begin{align*}
W = DW = T = G = 0, \quad & \text{at} \quad z = 0 \\
W = D^2W = T = G = 0, \quad & \text{at} \quad z = 1
\end{align*} \]  \tag{4.28}

The trial functions satisfying (4.28) are

\[ \begin{align*}
W_1 &= 2z^4 - 5z^3 + 3z^2, \\
T_1 &= z(1-z), \\
G_1 &= z(1-z)
\end{align*} \]  \tag{4.29}

Substituting (4.29) in equation (4.27) and performing the integration, we get

\[ R = \frac{-K_1}{30} \left[ (1 + N_1)K_2 + N_1^2K_3 + QK_4 \right] \frac{1}{a^2 * \left[ -K_5 + N_1 N_5 K_6 - R_1 K_7 K_5 \right]} \]  \tag{4.30}

where

\[ K_1 = (10 + a^2), \]
\[ K_2 = \frac{4536 + 19a^4 + 432a^2}{630}, \]
\[ K_3 = \frac{15876 + 169a^4 + 3276a^2}{(58800 + 5880a^2)N_3 + 11760N_1}, \]
\[ K_4 = \frac{12}{35}, \]
\[ K_5 = \frac{13}{420}, \]
\[ K_6 = \frac{3780 + 390a^2}{(126000 + 12600a^2)N_3 + 25200N_1}, \]
\[ K_7 = \frac{1}{840} \]

R attains its minimum value \( R_c \) at \( a = a_c \).

4.5. CRITICAL RAYLEIGH NUMBER FOR ADIABATIC BOUNDARIES:

We consider three cases,

i. Both boundaries are free (Free – Free)

ii. Both boundaries are rigid (Rigid – Rigid) and

iii. Lower boundary rigid and upper boundary free (Rigid – Free).

In all the above three cases, we take spin – vanishing boundary condition on microrotation.

4.5.1 Critical Rayleigh number when lower boundary is rigid and upper boundary is free:

The boundary conditions are
\[
W = DW = DT = G = 0, \quad at \quad z = 0
\]
\[
W = D^2W = DT = G = 0, \quad at \quad z = 1
\]  
(4.31)

The trial functions satisfying (4.31) are
Substituting (4.32) in equation (4.27) and performing the integration, we get

\[
R = \frac{-a^2 [(1 + N_1) K_1 + N_1^2 K_2 + QK_3]}{a^2 \left[ -K_4 + N_1 N_5 K_5 - R_1 K_6 \right]}.
\]

(4.33)

Where

\[
K_1 = \frac{4536 + 19a^4 + 432a^2}{630},
\]

\[
K_2 = \frac{15876 + 169a^4 + 3276a^2}{(58800 + 5880a^2)N_3 + 11760N_1},
\]

\[
K_3 = \frac{12}{35},
\]

\[
K_4 = \frac{9}{400},
\]

\[
K_5 = \frac{11340 + a^2 1170}{(504000 + 50400a^2)N_3 + 100800N_1},
\]

\[
K_6 = \frac{3}{2400}
\]

R attains its minimum value R_c at a = a_c

4.6 Marangoni magneto-convection in a Micropolar fluid with Internal Heat generation

The convection induced by the variation of surface tension with temperature in the absence of buoyancy force is generally referred, in the literature as Marangoni convention. In this section, we study Marangoni
magneto convection in a micropolar fluid with internal heat generation. The linear theory is studied, using the single term Galerkin expansion procedure. The basic equation are

\[ \nabla \cdot \vec{q} = 0, \quad (4.34) \]

\[ \rho_o \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla P + (2\zeta + \eta)\nabla^2 \vec{q} + \zeta \nabla \times \vec{\omega} + \mu_m (\vec{H} \cdot \nabla) \vec{H}, \quad (4.35) \]

\[ \rho_o I \left[ \frac{\partial \vec{\omega}}{\partial t} + (\vec{q} \cdot \nabla) \vec{\omega} \right] = (\lambda' + \eta')\nabla(\nabla \cdot \vec{\omega}) + (\eta'\nabla^2 \vec{\omega}) + \zeta (\nabla \times \vec{q} - 2\vec{\omega}), \quad (4.36) \]

\[ \frac{\partial T}{\partial t} + \left( \vec{q} - \frac{\beta}{\rho_o C_v} \nabla \times \vec{\omega} \right) \nabla T = k\nabla^2 T + Q \quad (4.37) \]

\[ \frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} = (\vec{H} \cdot \nabla) \vec{q} + \gamma_m \nabla^2 \vec{H}, \quad (4.38) \]

\[ \rho = \rho_o [1 - \alpha (T - T_o)]. \quad (4.39) \]

The basic equation (4.34) to (4.39) are solved subjected to containment conditions appropriate for rigid and thermally perfect conducting wall on the underside and by a free surface on the upper side. This free surface is adjacent to a non-conducting medium and subject to a constant heat flux (i.e. adiabatic). Further, the no-spin boundary conditions is assumed for microrotation. Since the shear stress for a micropolar fluid is no different from that of classical fluids, the boundary condition for the flat free boundaries used by Nield (1966) in respect of Newtonian fluids are appropriate for micropolar fluid also.
Following the analysis of section (4.1 and 4.2) we get the non-dimensional form of the basic equation after eliminating \( H_z \) in the form:

\[
-Ra^2 T + (1+N_1)(D^2-a^2)^2 W + N_1(D^2-a^2)G - QD^2 W = 0 \quad (4.40)
\]

\[
N_3(D^2-a^2)G - N_1(D^2-a^2)W - 2N_1 G = 0 \quad (4.41)
\]

\[
(D^2-a^2)T + W - N_5 G - R_1 \left( \frac{1}{2} - Z \right) W + R_1 N_5 \left( \frac{1}{2} - Z \right) G = 0 \quad (4.42)
\]

Where

\[
D = \frac{d}{dz}
\]

The equation (4.38) to (4.40) are solved subject to the following boundary conditions

\[
\begin{align*}
W = D W = T = G &= 0 \quad \text{at} \quad z = 0 \\
W = D^2 W + a^2 MT = DT &= G = 0 \quad \text{at} \quad z = 1
\end{align*}
\quad (4.43)
\]

where:

M is the Marangoni number. The condition on \( G \) is the spin-vanishing boundary condition.

We now use the single term Galerkin expansion to find the critical eigenvalue \( M_c \), multiplying equation (4.40) by \( W \), equation (4.41) by \( G \) and equation (4.42) by \( T \), integrating the resulting equation by parts with respect to \( z \) from 0 to 1, using boundary condition (4.43) and taking \( W = AW_1, G = BG_1 \) and \( T = CT_1 \) in which A, B and C are constants and \( W_1, G_1 \) and \( T_1 \) are trial functions. This procedure yields the following equation for the Marangoni number M:
\[ M = \left[ \frac{(1 + N_1)M_8X_6 + X_7M_8}{a^2 * X_8} \right] \]

S(4.44)

Where

\[ M_1 = \langle W_1T_1 \rangle \]
\[ M_2 = \langle G_1(D^2 - a^2)W_1 \rangle \]
\[ M_3 = \langle G_1T_1 \rangle \]
\[ M_4 = R_I \langle (1/2 - Z)W_1T_1 \rangle \]
\[ M_5 = \langle G_1(D^2 - a^2)G_1 \rangle \]
\[ M_6 = G_1^2 \]
\[ M_7 = \langle \left( \frac{1}{2} - Z \right)G_1T_1 \rangle \]
\[ M_8 = \langle T_1(D^2 - a^2)T_1 \rangle \]
\[ M_9 = \langle (D^2W_1)^2 \rangle \]
\[ M_{10} = \langle (DW_1)^2 \rangle \]
\[ M_{11} = \langle W_1^2 \rangle \]
\[ M_{12} = \langle W_1(D^2 - a^2)G_1 \rangle \]
\[ M_{13} = \langle W_1D^2W_1 \rangle \]

\[ X_1 = M_2M_3 \]
\[ X_2 = N_3M_5 - 2N_1M_6 \]
\[ X_3 = M_2M_7 \]
\[ X_4 = M_2M_{12} \]
\[ X_5 = -M_1 + N_5N_1X_1 / X_2 + R_I M_4 - R_I N_5N_1X_3 / X_2 \]
\[ X_6 = M_9 + 2M_{10}a^2 + M_{11}a^{11} \]
\[ X_7 = N_1^2 X_4 / X_2 - QM_{13} \]
\[ X_8 = 2(1 + N_1)X_5 \]
In the equation (4.44), \(\langle - - - \rangle\) denotes the integration with respect to \(z\) between \(z = 0\) and \(z = 1\). We note here that \(M\) in the equation (4.44) is a functional and the Euler – Lagrange equation for the extermination of \(M\) are equations (4.40) to (4.42).

The trail function satisfying (4.43) are

\[
\begin{align*}
W_1 &= z^2(1 - z^2), \\
T_1 &= z(2 - z), \\
G_1 &= z(1 - z).
\end{align*}
\]

such that they satisfy all the boundary conditions (4.43) except the one given by \(D^2W + a^2MT = 0\) at \(z = 1\), but the residual from this is included in the residual from the differential equation. Substituting (4.45) in equation (4.44) and performing the integration, we get

\[
M = \frac{(1 + N_1)K_1K_2 - \left[ N_1^2K_3 + QK_4 \right]K_1}{a^2 \left[ (1 + N_1)\left[ -K_5 + N_1N_5K_6 + R_1N_1N_5K_7 \right] \right]} \tag{4.46}
\]

where

\[
K_1 = \frac{20 + 8a^2}{15},
\]

\[
K_2 = \frac{5292 + 264a^2 + 8a^2}{315}
\]

\[
K_3 = \frac{12544 + 2464a^2 + 121a^4}{N_3(58800 + 5880a^2) + N_111760},
\]

\[
K_4 = \frac{44}{105},
\]

\[
K_5 = \frac{23}{105},
\]
\[ K_6 = \frac{1568 + 154a^2}{N_3(8400 + 840a^2) + N_1 1680}, \]
\[ K_7 = \frac{224 + 22a^2}{N_3(16800 + 1680a^2) + N_1 3360} \]

M attains its minimum value \( M_c \) at \( a = a_c \)

4.7 Rayleigh – Bénard – Marangoni magneto convection in a micropolar fluid with internal heat generation.

In this section, we discuss the magnetic field on the onset of convection driven by both buoyancy force and the variation of surface tension force. The linear theory is studied, using the single term Galerkin expansion procedure.

The basic equations are

\[ \nabla \vec{q} = 0, \quad (4.47) \]

\[ \rho_o \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla P - \rho g \kappa + (2 \zeta + \eta) \nabla^2 \vec{q} + \zeta \nabla \times \vec{\omega} + \mu_m (\vec{H} \cdot \nabla) \vec{H}, \quad (4.48) \]

\[ \rho_o I \left[ \frac{\partial \vec{\omega}}{\partial t} + (\vec{q} \cdot \nabla) \vec{\omega} \right] = (\lambda' + \eta') \nabla (\nabla \cdot \vec{\omega}) + (\eta' \nabla^2 \vec{\omega}) + \zeta (\nabla \times \vec{q} - 2 \vec{\omega}), \quad (4.49) \]

\[ \frac{\partial T}{\partial t} + \left( \vec{q} - \frac{\beta}{\rho_o C_v} \nabla \times \vec{\omega} \right) \nabla T = -\nabla \vec{Q}, \quad (4.50) \]

\[ \frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} = (\vec{H} \cdot \nabla) \vec{q} + \gamma_m \nabla^2 \vec{H}, \quad (4.51) \]
\[ \rho = \rho_o [1 - \alpha (T - T_o)]. \] (4.52)

Where:
\[ \tilde{\omega}_1 = \frac{1}{2} \nabla \times \tilde{q}, \quad \tilde{Q} \] is the heat flux vector and \( \tau \) is the constant relation time.

The basic equation (4.47) to (4.52) are solved subjected to containment conditions discussed in section (4.6).

Following the analysis of section (4.1 and 4.2) we get the non-dimensional form of the basic equation after eliminating \( H_z \) in the form:

\[-Ra^2 T + (1 + N_1) (D^2 - a^2)^2 W + N_1 (D^2 - a^2) G - QD^2 W = 0 \] (4.53)

\[ N_3 (D^2 - a^2) G - N_1 (D^2 - a^2) W - 2 N_1 G = 0 \] (4.54)

\[(D^2 - a^2) T + W - N_5 G - R_I \left( \frac{1}{2} - Z \right) W + R_I N_5 \left( \frac{1}{2} - Z \right) G = 0 \] (4.55)

Where
\[ D = \frac{d}{dz} \]

The equation (4.53) to (4.55) are solved subject to the following boundary conditions

\[ W = D W = T = G = 0 \quad \text{at} \quad z = 0 \]\n\[ W = D^2 W + a^2 MT = DT = G = 0 \quad \text{at} \quad z = 1 \] (4.56)

Where:
M is the Marangoni number. The condition on G is the spin-vanishing boundary condition.

We now use the single term Galerkin expansion to find the critical eigenvalue $M_c$, multiplying equation (4.53) by $W$, equation (4.54) by $G$ and equation (4.55) by $T$, integrating the resulting equation by parts with respect to $z$ from 0 to 1, using boundary condition (4.56) and taking $W = AW_1$, $G = BG_1$ and $T = CT_1$ in which $A$, $B$ and $C$ are constants and $W_1$, $G_1$ and $T_1$ are trial functions. This procedure yields the following equation for the Marangoni number $M$:

$$M = \frac{-Ra^2X_5M_1 + (1+N)M_8X_6 + X_7M_8}{a^2* X_8}$$

(4.57)

Where

$M_1 = \langle W_1T_1 \rangle$

$M_2 = \langle G_1(D^2 - a^2)W_1 \rangle$

$M_3 = \langle G_1T_1 \rangle$

$M_4 = R_1\left(\frac{1}{2} - Z\right)W_1T_1$

$M_5 = \langle G_1(D^2 - a^2)G_1 \rangle$

$M_6 = G_1^2$

$M_7 = \left(\frac{1}{2} - Z\right)G_1T_1$

$M_8 = \langle T_1(D^2 - a^2)T_1 \rangle$

$M_9 = \langle (D^2W_1)^2 \rangle$

$M_{10} = \langle (DW_1)^2 \rangle$

$M_{11} = \langle W_1^2 \rangle$
\[ M_{12} = \langle w_1 (D^2 - a^2) g_1 \rangle \]
\[ M_{13} = \langle w_1 D^2 w_1 \rangle \]
\[ X_1 = M_2 M_3 \]
\[ X_2 = N_3 M_5 - 2N_1 M_6 \]
\[ X_3 = M_2 M_7 \]
\[ X_4 = M_2 M_{12} \]
\[ X_5 = -M_1 + N_5 N_1 X_1 / X_2 + R_1 M_4 - R_1 N_5 N_1 X_3 / X_2 \]
\[ X_6 = M_9 + 2M_{10} a^2 + M_{11} a^{11} \]
\[ X_7 = N_1^2 X_4 / X_2 - QM_{13} \]
\[ X_8 = 2(1 + N_1) X_5 \]

For \( R = 0 \), the equation (4.57) reduces to the equation (4.44), the Marangoni number for Marangoni convection.

The chosen trial functions are

\[
\begin{align*}
  W_1 &= z^2 (1-z^2), \\
  T_1 &= z(2-z), \\
  G_1 &= z(1-z).
\end{align*}
\]

This satisfy all the boundary conditions (4.56) except the one given by \( D^2 W + a^2 MT = 0 \) at \( z = 1 \), but the residual from this is included in the residual from the differential equation. Substituting (4.58) in equation (4.57) and performing the integration, we get

\[
M = \frac{-Ra^2 \left[ -K_1 + N_1 N_5 K_2 + R_1 N_1 N_5 K_3 \right] K_1 - (1 + N) K_4 K_5 \left[ N_1^2 K_6 + QK_7 \right] K_4}{a^* \left[ -K_1 + N_1 N_5 K_2 + R_1 N_1 N_5 K_3 \right]} \tag{4.59}
\]

Where

\[
K_1 = \frac{23}{210},
\]
\[ K_2 = \frac{784 + 77a^2}{N_3(8400 + 840a^2) + N_11680}, \]
\[ K_3 = \frac{112 + 11a^2}{N_3(16800 + 1680a^2) + N_13360}, \]
\[ K_4 = \frac{20 + 8a^2}{15}, \]
\[ K_5 = \frac{5992 + 264a^2 + 8a^4}{315}, \]
\[ K_6 = \frac{12544 + 2464a^2 + 121a}{N_3(58800 + 5880a^2) + N_111760}, \]
\[ K_7 = \frac{44}{105} \]

M attains its minimum value \( M_c \) at \( a = a_c \)
Table 1: value of critical wave number $a_c^2$ for different value $N_1, N_3, N_5, R_I, Q$.

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Table 2: value of critical wave number $a_c^2$ for different value of $N_1, N_3, N_5, R_I, R, Q$.

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Table 3: Value of critical wave number $a^2_c$ for different values of $N_1, N_3, N_5, R, R, R, Q, M_c$ \[ N_1 = 0.1, N_3 = 2.0, N_5 = 1.0 \]

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Fig (1) Schematic diagram of Rayleigh-Benard Marangoni situation for a fluid with suspended particles.
Fig. 2. Plot of Critical Rayleigh number $R_C$ versus $N_1$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid-free isothermal boundaries.

N3=2.0, N5=1.0
1) R1=0, Q=0
2) R1=0, Q=50
3) R1=1, Q=50
4) R1=3, Q=50
5) R1=-3, Q=50
6) R1=3, Q=25
Fig. 3. Plot of Critical Rayleigh number $R_C$ versus $N_3$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid-free Isothermal Boundaries.
Fig. 4. Plot of Critical Rayleigh numbers $R_C$ versus $N_5$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid-free Isothermal Boundaries.
Fig. 5. Plot of Critical Rayleigh number $R_c$ versus $N_1$ for different values of Chandrasekhar number $Q$ Internal Rayleigh number $R_I$ for rigid-free Adiabatic Boundaries.
Fig. 6. Plot of Critical Rayleigh number $R_C$ versus $N_3$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid- free Adiabatic Boundaries.

$N_1=0.1, N_5=1.0$

1) $R_I=0, Q=0$  
2) $R_I=0, Q=50$  
3) $R_I=1, Q=50$  
4) $R_I=3, Q=50$  
5) $R_I=3, Q=25$  
6) $R_I=-3, Q=50$
Fig. 7. Plot of Critical Rayleigh number $R_C$ versus $N_5$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid- free Adiabatic Boundaries.
Fig. 8. Plot of Critical Marangoni number $M_C$ versus $N_1$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid- free Isothermal Boundaries.
Fig. 9. Plot of Critical Marangoni number $M_c$ versus $N_3$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid- free Isothermal Boundaries.
Fig. 10. Plot of Critical Marangoni number $M_c$ versus $N_5$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh number $R_I$ for rigid-free Isothermal Boundaries.
Fig. 11. Plot of Critical Marangoni number $M_C$ versus $N_1$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh Number $R_I$ for rigid-free Isothermal Boundaries
Fig. 12. Plot of Critical Marangoni number $M_C$ versus $N_3$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh Number $R_I$ for rigid-free Isothermal Boundaries.
$N_1 = 0.1, N_3 = 2.0$

1) $R_I = 0, Q = 0, R = 0$
2) $R_I = 0, Q = 50, R = 0$
3) $R_I = 1, Q = 50, R = 0$
4) $R_I = 3, Q = 50, R = 0$
5) $R_I = 3, Q = 25, R = 0$
6) $R_I = -3, Q = 50, R = 0$
7) $R_I = 3, Q = 25, R = 100$
8) $R_I = 3, Q = 25, R = 300$

Fig. 13. Plot of Critical Marangoni number $M_C$ versus $N_5$ for different values of Chandrasekhar number $Q$ and Internal Rayleigh Number $R_I$ for rigid- free Isothermal Boundaries.
CHAPTER V

RESULTS, DISCUSSIONS AND CONCLUSION

In this dissertation, the effect of internal heat generation is investigated on the stability of Rayleigh – Bénard – Marangoni convection in micropolar fluid with the objective of understanding the control (i.e., suppress or augment) of convective instability which is important in many engineering and energy storage applications. The results of the eigen value problem is solved by Galerkin method.

Prior to applying the method to the complete problem, test computations have been performed in the absence of internal heat generation and the results are compared with Siddheshwar and Pranesh (1998). The critical Rayleigh number $R_c$ obtained for different values of $R_I$ and $N_1 = 0$ are compared with Sparrow et al (1964).

Figure (2) is the plot of critical Rayleigh number $R_c$ versus the coupling number $N_1$ for rigid free isothermal boundary combinations for different values of Chandrashekar number $Q$ and internal heat source parameter $R_I$. It is observed that as $N_1$ increases, $R_c$ also increases. Increase in $N_1$ indicates the increase in the concentration of the microelements. These microelements consume the greater part of the energy in developing gyrational velocity and as a result the onset of convection is delayed. From this we conclude that an increase in $N_1$ is to stabilise the system. From the figure it is observed that $R_I$ that represents internal heat source parameter has a destabilising influence and the effect arising due to internal heat generation is to augment convection. It is also observed that the increase in $Q$ increases the $R_c$ from this we conclude that the $Q$ has stabilizing effect on the system.
When the magnetic field strength permeating the medium is considerably strong, it induces viscosity into the fluid, and the magnetic lines are distorted by convection. Then these magnetic lines hinder the growth of disturbances, leading to the delay in the onset of instability. However, the viscosity produced by the magnetic field lessens the rotation of the fluid particles, thus controlling the stabilizing effect of $N_1$.

Figure (3) is the plot of critical Rayleigh number $R_c$ versus the couple stress parameter $N_3$ for rigid free isothermal boundary combinations for different values of Chandrashekar number $Q$ and internal heat source parameter $R_I$. We note that the role played by the shear stress in the conservation of linear momentum is played by couple stress in the conservation of angular momentum equation. It is observed that as $N_3$ increases $R_c$ decreases. Because, when $N_3$ increases the couple stress of the fluid increases, this increase in the couple stress causes the micro-rotation to decrease. Therefore, increase in $N_3$ destabilises the system. From the figure it is observed that decrease in $R_c$ is significant for lower values of $N_3$ and at higher values, the dip in $R_c$ is insignificant. From this we conclude that couple stresses are operative at only small values of $N_3$. It is also observed that the increase in $R_I$ decreases the $R_c$.

Figure (4) is the plot of critical Rayleigh number $R_c$ versus the micropolar heat conduction parameter $N_5$ for rigid free isothermal boundary combinations for different values of Chandrashekar number $Q$ and internal heat source parameter $R_I$. When $N_5$ increases, the heat induced into the fluid due to these microelements also increases, thus reducing the heat transfer from bottom to top. The decrease in heat transfer is responsible for delaying the onset of instability. This result can also be anticipated because equation (4.4) clearly shows that the effect of the suspended particles is to deduct
from the velocity. We conclude that an increase in \( N_5 \) increases \( R_{sc} \) and thereby stabilises the system. It is also observed that the increase in \( R_I \) decreases the \( R_c \).

Figures (5) – (7) are the plot of critical Rayleigh number \( R_c \) with respect to adiabatic boundaries for different values of Chandrashekar number \( Q \) and internal heat source parameter \( R_I \). The results of these figures are quantitatively similar to isothermal case, except that the convection sets in at wave number, \( a = 0 \).

Figures (8) – (10) and figures (11) – (13) are the plot for Marangoni convection (\( R = 0 \)) and Rayleigh – Bénard – Marangoni convection (\( R \neq 0 \)) respectively. From these figures following observations are made:

1. It is observed that as \( Q \) increases \( M_c \) also increases. It is also observed that as \( N_1 \) increases, \( M_c \) also increases for small values of \( Q \). However, for very large values of \( Q \), the critical \( M_c \) is less than the Newtonian value. This result may possibly suggest a value of \( Q \) upto which the present theoretical study applies. Thus, an increase in the concentration of suspended particles is to stabilise the system along with the magnetic field.

2. The effect of \( N_3 \) on the system is very small compared to the effects of the other micropolar parameters.

3. The effect of \( N_5 \) is to stabilize the system. It is also clear that when the coupling between temperature and spin increases the system is more stable compared to the case when there is no coupling.
4. As $R$ becomes larger and larger, $M_c$ becomes smaller and smaller and ultimately we get the case of convection dominated by buoyancy force.

5. In the limit $N_1 \to 0$, we recover the results of Siddheshwar and Pranesh (1998) from the present study.

From the table (1) it is clear that with an increase in $R_i$, $M_c$ becomes zero for higher values of $R$ compared to Newtonian fluids.

From the table (2) it was found that the critical wave number for stationary convection is, in general, insensitive to the changes in the micropolar parameters but is influenced by the magnetic field. A strong magnetic field succeeds in inducing only the coupling parameter $N_1$ into influencing $a_c^2$. We also find that $a_c^2$ increases with magnetic field. This indicates that, the convection cell at onset diminishes in size with increasing magnetic field strength.

Following conclusions are drawn from the problem:

1. There are two different kinds of instability: the surface tension mode and the thermal mode. The internal heat generation in the fluid layer has a significant influence on both modes and is clearly a destabilizing factor to make the system more unstable.

2. Stationary convection is the preferred mode of instability in a micropolar fluid.
3. Rayleigh-Bénard convection in Newtonian fluids may be delayed by adding micron sized suspended particles.

4. The system becomes more unstable with an increase in the value of $R_I$. 
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