INVERSE DOMINATION NUMBER OF GRAPHS

Dissertation submitted in partial fulfillment of the requirements for the award of the degree of

MASTER OF PHILOSOPHY IN MATHEMATICS

By

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Research Guide

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2010

Srinivasa Ramanujan (1887-1927)

Indian Mathematician

Leonhard Euler (1707-1783)

Swiss Mathematician

“Father of Graph Theory”

DEDICATED TO MY PARENTS

C.NIGIDA IMMANUEL

&

MARY IMMANUEL

DECLARATION

I hereby declare that the dissertation entitled **”Inverse domination number of graphs”** has been undertaken by me for the award of M.Phil degree in Mathematics.I have completed this under the guidance of **Dr.Shivasharanappa Sigarkanti**, M.Sc, Ph.D.,Head of the Department, Department of Mathematics,Government Science College,Bangalore.

I also declare that this dissertation has not been submitted for the award of any Degree,Diploma,Associate-ship,Fellowship,etc., in this University or in any other University.

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CERTIFICATE

This is to certify that the dissertation submitted by Levi Chellson N. on the title "**Inverse domination number of graphs**" is a bonafide record of research work done by him during the academic year 2009-2010 under my guidance and supervision in partial fulfillment of Master of Philosophy in Mathematics.This dissertation has not been submitted for the award of any Degree,Diploma,Associate-ship,Fellowship,etc., in this University or in any other University.

Place: Bangalore Dr.Shivasharanappa Sigarkanti

Date: (Guide)

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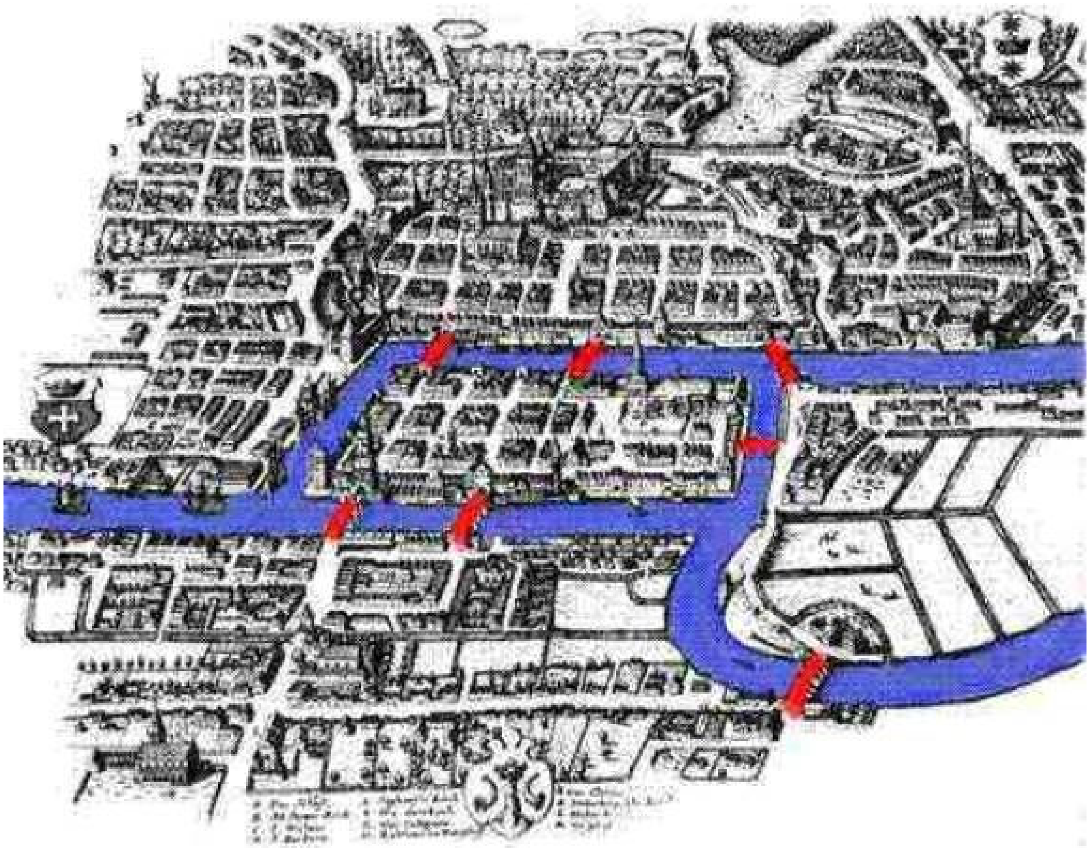
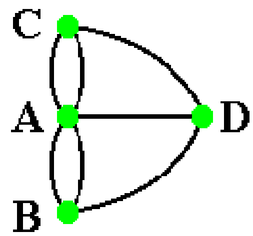
Chapter 1 INTRODUCTION

1.1 HISTORY OF GRAPH THEORY



The Konigsberg Bridge Problem:

Konigsberg (55.2 ^{o} North latitude and 22 ^{o} East longitude) was a city in Russia situated on the Pregel River,which served as the residence of the dukes of Prussia in the 16th century.Today,the city named Kaliningrad,is in Lithuania which recently separated from U.S.S.R.It serves as a major industrial and commercial centre of western Russia.The river Pregel flowed through the town,dividing it into four regions,as in the following picture.In the eighteenth century,seven bridges connected the four regions.The problem was to start from anyone of the land areas,walk across each bridge exactly once and return to the starting point.This problem was first solved in 1736 by the prolific Swiss Mathematician Leonhard Euler,who,as a consequence of his solution invented the branch of Mathematics now known as Graph Theory.Euler's solution consisted of representing the problem by a "graph" with the four regions represented by four vertices and the seven bridges by seven edges as follows:



Euler abstracted the problem by replacing each land area by a vertex and each bridge by an edge joining these vertices leading to a 'multi graph' as shown in the figure.The Konigsberg Bridge Problem is the same as the problem of drawing the above figure without lifting the pen from the paper and without retracing any line and coming back to the starting point.This problem was generalised and a necessary and sufficient condition for a graph to be so traversable has been obtained.

The Four Colour Problem:

One of the most famous problems in Graph Theory is the Four Colour Problem.The problem states that any map on a plane or on the surface of a sphere can be coloured with four colours in such a way that no two adjacent countries have the same colour.This problem can be translated as a problem in Graph Theory.We represent each country by a vertex and join two vertices by an edge if the countries are adjacent.The problem is to colour the vertices in such a way that adjacent vertices receive different colours.This problem was first posed in 1852 by Francis Guthrie,a post-graduate student at the

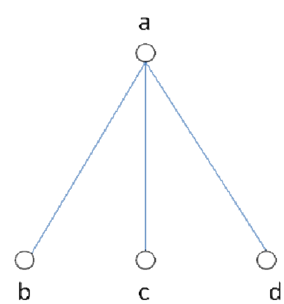
University College,London.This problem was finally proved by Appel and Haken in 1976 and they have used 400 pages of arguments and about 1200 hours of computer time on some of the best computers in the world to arrive at the solution.

1.2 GRAPHS

Graph:

A graph G consists of a pair (V(G),E(G)) where V(G) is a nonempty finite set whose elements are called points or vertices and E(G) is a set of unordered pairs of distinct elements of V(G).The elements of E(G) are called lines or edges of the graph G.A graph G with p vertices and q edges is called a (p,q) graph.

Example:



Here V(G)={a,b,c,d}.E(G)={{a,b},{a,c},{a,d}}.This graph G=(V,E) is a (4,3) graph.The number p is referred to as the order of G and the number q is referred to as the size of G.If G is a (p,q) graph then $p\geq1$ and $0\leq q\leq p(p-1)/2$.

Incidency:

If e={u,v}ϵE(G),the edge e is said to join the vertices u and v.We say that the vertices u and v are incident with the edge e.

Adjacency:

If e=uv we say that the vertices u and v are adjacent.If two distinct edges e and f have a common vertex v,we say that the edges e and f are adjacent.

1.3 TYPES OF GRAPHS

Loop**:**

A line joining a point to itself is called a loop.In other words a loop is an edge with identical ends.

Link:

An edge with distinct ends is called a link.

Finite graph:

A graph is finite if both its vertex set and edge set are finite.

Trivial graph:

A graph with just one vertex is called a trivial graph.

Nontrivial graph:

A graph with more than one vertex is called a nontrivial graph.

Empty graph:

A graph whose edge set is empty is called an empty graph.It is also called as a null graph or a totally disconnected graph.

Parallel edges:

If two or more edges of a graph have the same end vertices then these edges are called parallel edges.The parallel edges are also called multiple edges.

Simple graph:

A simple graph is one which has no loops and no parallel edges.

Multi graph:

If more than one edge joining two vertices are allowed,the resulting object is called a multi graph.The Konigsberg Bridge problem is an example of a multi graph.

Pseudo graph**:**

A multi graph in which loops are allowed is called a pseudo graph.

Planar graph**:**

A graph which can be represented in a plane and whose edges intersect only at their end vertices is called a planar graph.

Complete graph**:**

A simple graph in which each pair of distinct vertices is joined by an edge is called a complete graph.The complete graph on n vertices is denoted by K_n.K_3 (triangle) is an example of a complete graph.

Bipartite graph**:**

A bipartite graph is one whose vertex set can be partitioned into two subsets X and Y so that each edge has one end in X and one end in Y.Such a partition (X,Y) is called a bipartition of the graph.The cube is an example of a bipartite graph.

K-partite graph**:**

A K-partite graph is one whose vertex set can be partitioned into K subsets so that no edge has both ends in any one subset.

Complete bipartite graph**:**

A complete bipartite graph is a simple bipartite graph with bipartition (X,Y) in which each vertex of X is joined to each vertex of Y.If IXI=m and IYI=n,such a graph is denoted by $K_{m,n}$.$K_{3,3}$ is an example of a complete bipartite graph.

Complete K- partite graph**:**

A complete K-partite graph is one that is simple and in which each vertex is joined to every vertex that is not in the same subset.

K-cube:

The K-cube is the graph whose vertices are the ordered K-tuples of 0's and 1's; Two vertices being joined if and only if they differ exactly in one coordinate.The K-cube has 2^k vertices,$k2^{k-1}$edges and is bipartite.

Star:

The graph $K_{1,m}$ is called a star for $m\geq 1$.

1.4 GRAPH ISOMORPHISM

Isomorphism**:**

Two graphs G and H are said to be isomorphic (written G ~= H) if there are bijections θ:V(G)→V(H) and φ:E(G)→E(H) such that $\psi_G(e)=uv$ if and only if $\psi_H(\varphi(e)) = \theta(u)\theta(v)$;Such a pair (θ,φ) of mappings is called an isomorphism between G and H.

1.5 GRAPH AUTOMORPHISM

Automorphism:

An isomorphism of a graph G onto itself is called an automorphism of G.

Let Γ(G) denote the set of all automorphisms of G.Clearly the identity map i:V→V defined by i(v)=v is an automorphism of G so that i∈Γ(G).Further if α and β are automorphisms of G then α·β and α^{-1} are also automorphisms of G.Hence Γ(G) is a group and is called the automorphism group of G.

Complement**:**

The complement G^c of a simple graph G is the simple graph with vertex set V, two vertices being adjacent in G^c if and only if they are not adjacent in G.

Self complementary graph**:**

A simple graph G is self complementary if G is isomorphic to G^c.C_5 is the only cycle which is self complementary.All other cycles are not self complementary.P_4 is self complementary.Any path other than P_4 is not self complementary.

1.6 SUBGRAPHS

Subgraph:

A subgraph H of G is a graph having all its vertices and edges of G.

Proper subgraph**:**

If H is a subset of G,but H≠G,then we say that H is properly contained in G, and H is called a proper subgraph of G.

Supergraph**:**

If H is a subgraph of G,then G is called a supergraph of H.

Spanning subgraph (or spanning supergraph):

A spanning subgraph (or spanning supergraph) of G, is a subgraph (or supergraph) H with V(H)=V(G).

Underlying simple graph:

By deleting from G all loops and,for every pair of adjacent vertices,all but one link joining them,we obtain a simple spanning subgraph of G,called the underlying simple graph of G.

Induced subgraph:

Suppose that V' is a nonempty subset of V.The subgraph of G whose vertex set is V' and whose edge set is the set of those edges of G that have both ends in V' is called the subgraph of G induced by V' and is denoted by G[V'] or <V'>;We say that G[V'] is an induced subgraph of G.The removal of a vertex v from a graph G yields the subgraph G-v of G containing all the vertices of G except v and all the edges not incident with v.

Edge induced subgraph:

Suppose that E' is a nonempty subset of E.The subgraph of G whose vertex set is the set of ends of edges in E' and whose edge set is E' is called the subgraph of G induced by E' and is denoted by G[E'] or <E'>;We say that G[E']is an edge induced subgraph of G.The removal of an edge e from a graph G yields the subgraph G-e of G containing all the vertices of G and all the edges of G except e.If u and v are not adjacent vertices in G, then the addition of edge e=(u,v) yields the graph G+e with the vertex set V and edge set EU{e}.

Disjoint and edge disjoint subgraphs:

Let G_1 and G_2 be two subgraphs of G.We say that G_1 and G_2 are disjoint if they have no vertex in common,and edge disjoint if they have no edge in common.

Edge graph:

The edge graph of a graph G is the graph with vertex set E(G) in which two vertices are joined if and only if they are adjacent edges in G.

1.7 OPERATIONS ON GRAPHS

Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be two graphs with their intersection being empty.We define:

The union G_1UG_2 to be (V,E) where,$V=V_1UV_2$ and $E=E_1UE_2$.

The sum G_1+G_2 as G_1UG_2 together with all the edges joining vertices of V_1 to the vertices of V_2.

The product G_1xG_2 as having $V=V_1xV_2$ and $u=(u_1,u_2)$ and $v=(v_1,v_2)$ are adjacent if $u_1=v_1$ and u_2 is adjacent to v_2 in G_2 or u_1 is adjacent to v_1 in G_1 and $u_2=v_2$.

The composition $G_1[G_2]$ as having $V=V_1xV_2$ and $u=(u_1,u_2)$ and $v=(v_1,v_2)$ are adjacent if u_1 is adjacent to v_1 in G_1 or ($u_1=v_1$ and u_2 is adjacent to v_2 in G_2).

Operation	Number of vertices	Number of edges
Union $G_1 U G_2$	p_1+p_2	q_1+q_2
Sum G_1+G_2	p_1+p_2	$q_1+q_2+p_1p_2$
Product G_1xG_2	p_1p_2	$p_1q_2+p_2q_1$
Composition $G_1[G_2]$	p_1p_2	$p_1q_2+p_2^2q_1$

1.8 VERTEX DEGREES

Degree:

The degree $d_G(v)$ of a vertex v in G is the number of edges of G incident with v, each loop counting as two edges.A vertex of degree 0 is called an isolated vertex.A vertex of degree 1 is called an end vertex or pendant vertex.

Theorem 1.1:

The sum of the degrees of the vertices of a graph G is twice the number of edges.(i.e.) $\sum_{v\epsilon V} d_G(v)=2q$.

Corollary 1.1:

In any graph,the number of vertices of odd degree is even.

Minimum degree,Maximum degree:

For any graph G,we define

$\delta(G)=\min\{\deg v/v\epsilon V(G)\}$,$\Delta(G)=\max\{\deg v/v\epsilon V(G)\}$.

Regular graph:

If all the vertices of G have the same degree r then $\delta(G)=\Delta(G)=r$ and in this case G is called a regular graph of degree r.A graph G is k-regular if $d_G(v)=k$ for all $v \in V$.A regular graph is one that is k-regular for some k.Complete graphs,complete bipartite graphs,and k-cubes are regular.A regular graph of degree 3 is called a cubic graph.The complete graph K_p is regular of degree p-1.

1.9 DEGREE SEQUENCES

Partition:

A partition of a nonnegative integer n,is a finite list of nonnegative integers,with sum n.

For example take n=4.Then P={4,3+1,2+2,2+1+1,1+1+1+1} is a required partition.

Degree sequence:

If G has vertices $v_1,v_2,\ldots,v_n$ the sequence $(d(v_1),d(v_2),\ldots,d(v_n))$ is called a degree sequence of G.

Graphical sequence:

A sequence $d=(d_1,d_2,\ldots,d_n)$ is graphic if there is a simple graph with degree sequence d.

Theorem 1.2:

A partition $P=(d_1,d_2,\ldots,d_p)$ of an even number into p parts with p-

$1 \geq d_1 \geq d_2 \geq \ldots \geq d_p$ is graphical iff the modified partition $P' = (d_2-1, d_3-1, \ldots, d_{d1+1}-1, d_{d1+2}-1, \ldots, d_p)$ is graphical.

Theorem 1.3 (Erdos and Gallai 1960):

If a partition $P=(d_1,d_2,\ldots,d_p)$ with $d_1 \geq d_2 \geq \ldots \geq d_p$ is graphical,then

$\sum^{p}_{i=1} d_i$ is even and $\sum^{k}_{i=1} d_i \leq k(k-1) + \sum^{p}_{i=k+1} \min\{k, d_i\}$ for $1 \leq k \leq p$.

1.10 PATHS AND CYCLES

Walk:

A walk in a graph G is a finite sequence $W=v_0e_1v_1e_2v_2\ldots v_{k-1}e_kv_k$ whose terms are alternatively vertices and edges such that each edge e_i is incident with v_{i-1} and v_i.We say that the above walk W is a v_0-v_k walk or a walk from v_0 to v_k.

Origin and terminus of the walk:

The vertex v_0 is called the origin of the walk W and the vertex v_1 is called the terminus of the walk W.

Internal vertices of the walk:

The vertices $v_1,\ldots,v_{k-1}$ in the above walk W are called the internal vertices of the walk.

Length of the walk:

The integer k, the number of edges in the walk,is called the length of the walk W.

Trivial walk:

A walk having no edges is called a trivial walk.

Closed walk:

Given two vertices u and v of a graph G,a u-v walk is called closed if u=v.

Open walk:

Given two vertices u and v of a graph G,a u-v walk is called open if u≠v.

Trail:

If the edges $e_1,e_2,...,e_k$ of the walk $W=v_0e_1v_1e_2v_2...v_{k-1}e_kv_k$ are distinct,then W is called a trail.

Path:

If the vertices $v_0,v_1,...,v_k$ of the walk $W=v_0e_1v_1e_2v_2...v_{k-1}e_kv_k$ are distinct,then W is called a path.Clearly any two paths with the same number of vertices are isomorphic.A path with n vertices is denoted by P_n.P_n has length n-1.

Cycle:

A closed trail whose origin and internal vertices are distinct is called a cycle.A cycle of length k is called a k-cycle.A k-cycle is odd or even according as k is odd or even.A 3-cycle is called a triangle.Clearly any two cycles of the same length are isomorphic.A cycle with n vertices is denoted by C_n.

Theorem 1.4:

A graph is bipartite if and only if it contains no odd cycle.

Girth of a graph:

The girth of a graph G is the length of a shortest cycle in G;If G has no cycles we define the girth of G to be infinite.

Distance between u and v in G:

The distance between u and v in G,denoted by $d_G(u,v)$,is the length of a shortest (u ,v) path in G;If there is no path connecting u and v we define $d_G(u,v)$ to be infinite.

Diameter of G:

The diameter of G denoted by diam(G),is the maximum distance between two vertices of G.

Circumference of G:

The circumference of a graph G is defined to be the length of its shortest cycle.

APPLICATIONS OF GRAPHS AND SIMPLE GRAPHS

The Shortest Path Problem:

Given a railway network connecting various towns,determine a shortest route between two specified towns in the network.With each edge e of G let there be associated a real number w(e),called its weight.Then G,together with these weights on its edges,is called a weighted graph.Our main aim is to find,in a weighted graph,a path of minimum weight connecting two vertices u and v.The weights represent distances by rail between directly linked towns,and are therefore nonnegative.Such a path of minimum weight can be found out using an algorithm known as Dijkstra's algotithm.This algorithm was first discovered by Dijkstra (1959).

Sperner's Lemma:

Sperner's Lemma concerns the decomposition of a simplex (line segment, triangle,tetrahedron) into simplices.For the sake of simplicity we shall deal with the two-dimensional case.Let T be a closed triangle in the plane.A subdivision of T into a finite number of smaller triangles is said to be simplicial if any two intersecting triangles have either a vertex or a whole side in common.Suppose that a simplicial subdivision of T is given.Then a labeling of the vertices of triangles in the subdivision in three symbols 0,1 and 2 is said to be proper if:The three vertices of T are labelled 0,1,2 (in any order),and for $0 \leq i < j \leq 2$,each vertex on the side of T joining vertices labelled i and j is labelled either i or j.We call a triangle in the subdivision whose vertices receive all three labels a distinguished triangle.The lemma says that,every properly labelled simplicial subdivision of a triangle has an odd number of distinguished triangles.

1.11 CONNECTIVITY

Connectedness:

Two vertices u and v of a graph G are said to be connected if there is a (u,v) path in G.

Disconnectedness:

Two vertices u and v of a graph G are said to be disconnected if there does not exist a (u,v) path in G.

Component:

The subgraphs $G[V_1],G[V_2],...,G[V_n]$ are called the components of G.If G has exactly one component,G is connected;Otherwise G is disconnected.

Vertex Cut:

A vertex cut of G is a subset V' of V such that G-V' is disconnected.A k-vertex cut is vertex cut of k elements.The only graphs which do not have vertex cuts are those that contain complete graphs as spanning subgraphs.

Edge cut:

An edge cut of G is a subset of E of the form [S,S'] where S is a nonempty proper subset of V.A k-edge cut is an edge cut of k elements.If G is nontrivial and E' is an edge cut of G,then G-E' is disconnected.

Bond:

A minimal edge cut of G is called a bond.

Cut vertex:

A vertex v of G is a cut vertex if E can be partitioned into two nonempty subsets E_1 and E_2 such that $G[E_1]$ and $G[E_2]$ have just the vertex v in common.If G is loopless and nontrivial,then v is a cut vertex of G if and only if $\omega(G-v)>\omega(G)$.

Cut edge:

A cut edge of G is an edge e such that $\omega(G-e)>\omega(G)$.It is also called a bridge.

k-connected:

A graph G is said to be k-connected if the removal of any set of less than k vertices does not disconnect the graph.

k-edge connected:

A graph G is said to be k-edge connected if the removal of any set of less than k edges does not disconnect the graph.

Connectivity $\kappa(G)$:

The connectivity $\kappa(G)$ of a graph G is the minimum k for which G has a k-vertex cut.Otherwise we define $\kappa(G)$ to be v-1.

$\kappa(G)=0$ if G is either trivial or disconnected.

G is said to be k-connected if $\kappa(G)\geq k$.

All nontrivial connected graphs are 1-connected.

Edge connectivity $\kappa'(G)$:

The edge connectivity $\kappa'(G)$ of a graph G is the minimum k for which G has a k-edge cut.

$\kappa'(G)=0$ if G is either trivial or disconnected.

$\kappa'(G)=1$ if G is a connected graph with a cut edge.

G is said to be k-edge connected if $\kappa'(G)\geq k$.

All nontrivial graphs are 1-edge connected.

Theorem 1.5:

For any graph G,$\kappa(G)\leq\kappa'(G)\leq\delta(G)$.

Block:

A connected graph that has no cut vertices is called a block.Every block with atleast three vertices is 2-connected.A block of a graph is a subgraph that is a block and is maximal with respect to this property.

Internally disjoint paths:

A family of paths in G is said to be internally disjoint,if no vertex of G is an internal vertex of more than one path of the family.

Theorem 1.6 (Whitney 1932):

A graph G with $v\geq3$ is 2-connected,if and only if any two vertices of G are connected by at least two internally disjoint paths.

Corollary 1.6.1:

If G is 2-connected,then any two vertices of G lie on a common cycle.

Subdivision of an edge:

An edge is said to be subdivided when it is deleted and replaced by a path of length two connecting its ends,the internal vertex of this path being a new vertex.

κ-critical:

A nonempty graph G is κ-critical if,for every edge e,$\kappa(G-e)>\kappa(G)$.

APPLICATIONS OF CONNECTIVITY

Construction Of Reliable Communication Networks:

If we think of a graph as representing a communication network,the connectivity (or edge connectivity) becomes the smallest number of communications (or communication links) whose breakdown would jeopardise communication in the system.The higher the connectivity and edge connectivity the more reliable is the network.

1.12 TREES

Acyclic graph:

A connected graph that contains no cycles is called an acyclic graph.

Leaf:

A leaf is a vertex of degree one.

Tree:

A connected acyclic graph is called a tree.

Forest:

Collection of all acyclic graphs (trees) is called a forest.

Caterpillar:

A caterpillar is a tree T for which the removal of all end vertices leaves a path, which is called a spine of T.

Spanning tree:

A spanning tree of G is a spanning subgraph of G that is a tree.

Theorem 1.7:

In a tree,any two vertices are connected by a unique path.

Theorem 1.8:

If G is a tree,then q=p-1.

Theorem 1.9:

Let G be a tree with p-1 edges.Then the following statements are equivalent.

G is connected;G is acyclic;G is a tree;

Theorem 1.10:

An edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

Corollary 1.11.1:

Every connected graph has a spanning tree.

Corollary 1.11.2:

If G is connected,then $q \geq p-1$.

Complement:

If H is a subgraph of G,the complement of H in G,denoted by $H^c(G)$,is the subgraph G-E(H).

Cotree:

If G is connected,a subgraph of the form T^c,where T is a spanning tree,is called a cotree of G.

Theorem 1.12:

A vertex v of a tree G is a cut vertex of G if and only if $d(v)>1$.

Contraction of an edge:

An edge e of G is said to be contracted if it is deleted and its ends are identified.The resulting graph is denoted by $G \cdot e$.

Theorem 1.13:

If e is a link of G,then τ is the number of spanning trees and $\tau(G)= \tau(G-e)+\tau(G \cdot e)$.

Theorem 1.14 (Cayley):

$\tau(K_n)=n^{n-2.}$

Wheel:

A wheel is a graph obtained from a cycle by adding a new vertex and edges joining it to all the vertices of the cycle.The new edges are called the spokes of the wheel.Thus a wheel is nothing but K_1+C_{p-1}.

Tree graph:

The tree graph of a connected graph G is the graph whose vertices are the spanning trees $T_1,T_2,...,T_k$ of G,with v_i and v_j joined if and only if they have v-2 edges in common.

Eccentricity e(v):

The eccentricity e(v) of a vertex v in G is defined by $e(v)=\max\{d(u,v)/u\epsilon V(G)\}$.

Radius r(G):

The radius r(G) is defined by $r(G)=\min\{e(v)/v\epsilon V(G)\}$.

Central point:

v is called a central point if e(v)=r(G).

Centre:

The set of all central points is called the centre of G.Centre is the vertex with minimum eccentricity.

Theorem 1.15:

Every tree has a centre consisting of either one vertex or two adjacent vertices.

APPLICATIONS OF TREES

The Connector Problem:

A railway network connecting a number of towns is to be set up.Given the cost c_{ij} of constructing a direct link between towns v_i and v_j,design such a network to minimize the total cost of construction.This is known as the connector problem.By regarding each town as a vertex in a weighted graph with weights $w(v_iv_j)=c_{ij}$,it is clear that this problem is just that of finding,in a weighted graph G,a connected spanning subgraph of minimum weight. Moreover,since the weights represent costs,they are certainly nonnegative, and we may therefore assume that such a minimum weight spanning subgraph is a spanning tree T of G.A minimum weight spanning tree of a weighted graph will be called an optimal tree.This optimal tree can be found out using an algorithm known as Kruskal's algorithm.

1.13 EULER TOURS AND HAMILTON CYCLES

Tour:

A tour of G is a closed walk that traverses each edge of G at least once.

Euler tour:

An Euler tour is a tour which traverses each edge of G exactly once.

Eulerian graph:

A graph is Eulerian if it contains an Euler tour.The Konigsberg Bridge

Problem is an example of an Eulerian graph.

Theorem 1.16:

A nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

Corollary 1.16:

A connected graph has an Euler trail if and only if it has atmost two vertices of odd degree.

APPLICATIONS OF EULER TOURS

The Chinese Postman Problem:

In his job,a postman picks up mail at the post office,delivers it,and then returns to the post office.He must,of course,cover each street in his area at least once.Subject to this condition,he wishes to choose his route in such a way that he walks as little as possible.This problem is known as the Chinese Postman Problem,since it was first considered by a Chinese Mathematician Kuan (1962).The Chinese Postman Problem is just that of finding a minimum weight tour in a weighted connected graph with nonnegative weights.We shall refer to such a tour as an optimal tour.If G is Eulerian,then any Euler tour of G is an optimal tour because,an Euler tour is a tour that traverses each edge exactly once.This problem is easily solved by using an algorithm known as Fleury’s algorithm.This algorithm constructs an Euler trail,subject to one condition that,at any stage,a cut edge of the untraced subgraph is taken only if there is no alternative.

Hamilton path:

A path that contains every vertex of G is called a Hamilton path of G.

Hamilton cycle:

A Hamilton cycle of G is a cycle that contains every vertex of G.A spanning cycle in a graph G is called a Hamilton cycle.

Hamiltonian graph:

A graph is Hamiltonian if it contains a Hamilton cycle.The dodecahedron is an example of a Hamiltonian graph.The Herschel graph is an example of a Nonhamiltonian graph,because it is bipartite and it contains an odd number of vertices.

Theorem 1.17(Dirac 1952):

If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$,then G is Hamiltonian.

Closure c(G):

The closure of G is the graph obtained from G by recursively joining pairs of nonadjacent vertices whose degree sum is atleast v until no such pair remains.It is denoted by c(G).

Lemma 1.18:

c(G) is well defined.

Theorem 1.19 (Chavatal 1972):

Let G be a simple graph with degree sequence $(d_1,d_2,...,d_v)$ with

$d_1 \leq d_2 \leq \ldots \leq d_v$ and $v \geq 3$.Suppose that there is no value of m less than v/2 for which $d_m \leq m$ and $d_{v-m} \leq v-m$.Then G is Hamiltonian.

Hamilton-connected:

A graph G is Hamilton-connected if every two vertices of G are connected by a Hamilton path.

Hypohamiltonian graph:

A graph G is Hypohamiltonian if G is not Hamiltonian but G-v is Hamiltonian for every $v \epsilon V$.The Petersen graph is an example of a Hypohamiltonian graph.

Hypotraceable graph:

A graph G is hypotraceable if G has no Hamilton path but G-v has a Hamilton path for every $v \epsilon V$.The Thomassen graph is an example of a hypotraceable graph.

APPLICATIONS OF HAMILTON CYCLES

The Travelling Salesman Problem:

A travelling salesman wishes to visit a number of towns and then return to his starting point.Given the travelling times between the towns,how should he plan his itinerary so that he visits each town exactly once and travels in all for as short a time as possible?This is known as the Travelling Salesman Problem.Our main aim of this problem is to find a minimum weight Hamilton cycle in a weighted complete graph.We call such a cycle an optimal cycle.Such an optimal cycle can be found out by using an algorithm known as the Kruskal’s algorithm.

1.14 INDEPENDENT SETS AND CLIQUES

Independent sets:

A subset S of V is called an independent set of G if no two vertices of S are adjacent in G.

Maximum independent set:

An independent set is maximum if G has no independent set S’ with IS’I>ISI.

Independence number β(G):

The number of vertices in a maximum independent set of G is called the independence number of G and it is denoted by β(G).

Edge independence number β’(G):

The number of edges in a maximum independent set of G is called the edge independence number of G and it is denoted by β’(G).

Covering:

A subset K of V such that every edge of G has at least one end vertex in K is called a covering of G.

Minimum covering:

A covering K is minimum if G has no covering K’ with IK’I<IKI.

Edge covering:

An edge covering of G is a subset L of E such that each vertex of G is an end vertex of some edge in L.

A graph G has an edge covering if and only if $\delta(G)>0$.

Covering number $\alpha(G)$:

The number of vertices in a minimum covering of G is called the covering number of G and it is denoted by $\alpha(G)$.

Edge covering number $\alpha'(G)$:

The number of edges in a minimum covering of G is called the edge covering number of G and it is denoted by $\alpha'(G)$.

Corollary 1.20:

$\alpha(G)+\beta(G)=p$.

Theorem 1.21 (Gallai 1959):

$\alpha'(G)+\beta'(G)=p$.

α-critical:

A graph is α-critical if $\alpha(G-e)>\alpha(G)$ for all $e \in E$.

β-critical:

A graph is β-critical if $\beta(G-e)<\beta(G)$ for all $e \in E$.

APPLICATIONS OF INDEPENDENT SETS

Theorem 1.22 (Schur 1916):

Let $(S_1,S_2,...,S_n)$ be any partition of the set of integers $\{1,2,...,r_n\}$.Then,for some i,S_i contains three integers x,y and z satisfying the equation x+y=z.

Clique:

A clique of a simple graph G is a subset S of V such that G[S] is complete.

Theorem 1.23 (Ramsey):

For any two integers $k\geq2$ and $l\geq2$,

$r(k,l)\leq r(k,l-1)+r(k-1,l)\rightarrow(1)$. Furthermore,if r(k,l-1) and r(k-1,l) are both even,then strict inequality holds in (1).

Theorem 1.24 (Erdös 1947):

$r(k,k)\geq2^{k/2}$.

APPLICATIONS OF CLIQUES

A Geometry Problem:

Theorem 1.25:

If $\{x_1,x_2,...,x_n\}$ is a set of diameter 1 in the plane,the maximum possible number of pairs of points at distance greater than 1/1.414 is $[n^2/3]$.Moreover,for each n,there is a set $\{x_1,x_2,...,x_n\}$ of diameter 1 with exactly $[n^2/3]$ pairs of points at distance greater than 1/1.414.

1.15 MATCHINGS

Matching:

Any set M of independent edges of a graph G is called a matching of G.If uv∈M, we say that the two end vertices u and v of the edge are matched under M.

M-saturated:

A vertex v that is saturated by a matching M is said to be M-saturated.

M-unsaturated:

A vertex v that is not saturated by a matching M is said to be M-unsaturated.

Perfect matching:

A matching M is called a perfect matching if every vertex of G is M-saturated.

Maximum matching:

A matching M is maximum if G has no matching M' with $|M'|>|M|$.

M-alternating path:

An M-alternating path in G is a path whose edges are alternatively in E\M and M.

M-augmenting path:

An M-augmenting path is an M-alternating path whose origin and terminus are both M-unsaturated.

Theorem 1.26 (Berge 1957):

A matching M in G is a maximum matching if and only if G contains no M-augmenting path.

k-factor:

A k-factor of G is a k-regular spanning subgraph of G.

k-factorable:

A graph G is k-factorable if there are edge disjoint k-factors $H_1,H_2,...,H_n$ such that $G=H_1UH_2U...UH_n$.

Neighbour set $N_G(S)$:

For any set S of vertices in G,we define the neighbour set of S in G to be the set of all vertices adjacent to vertices in S.This set is denoted by $N_G(S)$.

Theorem 1.27 (Hall 1935):

Let G be a bipartite graph with bipartition(X,Y).Then G contains a matching that saturates every vertex in X if and only if IN(S)I≥ISI,for all S which is a subset of X.

Corollary 1.27:

If G is a k-regular bipartite graph with k>0,then G has a perfect matching.

Lemma 1.28:

Let M be a matching and K be a covering such that IMI=IKI.Then M is a maximum matching and K is a minimum covering.

Theorem 1.28:

In a bipartite graph,the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

Theorem 1.29 (Tutte 1947):

G has a perfect matching if and only if o(G-S)≤ ISI for all S properly contained in V,where o(G) denotes the number of odd components of G.

APPLICATIONS OF MATCHINGS

The Personnel Assignment Problem:

In a certain company,n workers $X_1,X_2,...,X_n$ are available for n jobs $Y_1,Y_2,...,Y_n$, each worker being qualified for one or more of these jobs.Can all the men be assigned, one man per job,to jobs for which they are qualified?This is known as the Personnel Assignment Problem.We construct a bipartite graph G with bipartition (X,Y),where $X=\{x_1,x_2,...,x_n\}$ and $Y=\{y_1,y_2,..., y_n\}$,and x_i is joined to y_j,if and only if worker X_i is qualified for the job Y_j.The problem becomes one of determining whether or not G has a perfect matching.Our main aim of this problem is to find a maximum matching in the given bipartite graph.This maximum matching can be found out by using an algorithm known as the Hungarian Method.

The Optimal Assignment Problem:

In this problem,in addition,we wish to take into account the effectiveness of the workers in their various jobs (measured,perhaps,by profit to the company).In this case, one is interested in an assignment that maximizes the total effectiveness of the workers.The problem of finding such an assignment is known as the Optimal Assignment Problem.We consider a weighted complete bipartite graph with bipartition (X,Y),where $X=\{x_1,x_2,...,x_n\}$and $Y=\{y_1,y_2,...,y_n\}$,and edge x_iy_j has weight $w_{ij}=w(x_iy_j)$,the effectiveness of worker X_i in job Y_j.Our main aim of this problem is to find a maximum weight perfect matching in this weighted graph.We call such a matching as an optimal matching.This optimal matching can be found out by using an algorithm known as The Kuhn-Munkres algorithm which was first considered by Kuhn (1955) and Munkres (1957).

1.16 VERTEX COLOURINGS

k-vertex colouring:

A k-vertex colouring of a graph G is an assignment of k colours,1,2,…,k, to the vertices of G.The colouring is proper if no two distinct adjacent vertices have the same colour.

k-vertex colourable:

A graph G is k-vertex colourable if G has a proper k-vertex colouring. k-vertex colourable is also called as k-colourable.A graph is k-colourable if and only if its underlying simple graph is k-colourable.

Uniquely k-colourable:

A graph G is called uniquely k-colourable if any two proper k-colourings of G induce the same partition of V.

Chromatic number $\chi(G)$:

The chromatic number $\chi(G)$,of a graph G is the minimum k for which G is k-colourable.

k-chromatic:

If $\chi(G)=k$,G is said to be k-chromatic.

Critical graph:

A graph G is critical if $\chi(H)<\chi(G)$ for every proper subgraph H of G.Such graphs were first investigated by Dirac (1952).

k-critical:

A k-critical graph is one that is k-chromatic and critical.Every k-chromatic graph has a k-critical subgraph.

Theorem 1.30:

If G is k-critical,then $\delta(G)\geq k-1$.

Theorem 1.32:

In a critical graph,no vertex cut is a clique.

Theorem 1.33 (Brook 1973):

If G is a connected simple graph and is neither an odd cycle nor a complete graph, then $\chi(G)\leq\Delta(G)$.

Chromatic polynomial f(G,t):

The chromatic polynomial f(G,t) of a graph G is the number of different colourings of a labeled graph G that use t (or) fewer colours(like permutation and combination).

Theorem 1.34:

If G is a tree with n vertices then $f(G,t)=t(t-1)^{n-1}$.

APPLICATIONS OF VERTEX COLOURINGS

A Storage Problem:

A company manufactures n chemicals $C_1,C_2,...,C_n$.Certain pairs of these chemicals are incompatible and would cause explosions if brought into contact with each other.As a precautionary measure the company wishes to partition its warehouse into compartments, and store incompatible chemicals in different compartments.What is the least number of compartments into which the warehouse should be partitioned?This is known as the Storage Problem.We obtain a graph on the vertex set $\{v_1,v_2,...,v_n\}$ by joining two vertices v_i and v_j if and only if the chemicals C_i and C_j are incompatible.It is easy to see that the least number of compartments into which the warehouse should be partitioned is equal to the chromatic number of G.

1.17 EDGE COLOURINGS

k-edge colouring:

A k-edge colouring of a loopless graph G is an assignment of k colours,1,2,…,k, to the edges of G.The colouring is proper if no two distinct adjacent edges have the same colour.

k-edge colourable:

A graph G is k-edge colourable if G has a proper k-edge colouring.

Edge chromatic number $\chi'(G)$:

The edge chromatic number $\chi'(G)$,of a loopless graph G is the minimum k for which G is k-edge colourable.

k-edge chromatic:

If $\chi'(G)=k$,G is said to be k-edge chromatic.

Let ℓ be a given k-edge colouring of G.We shall denote by c(v) the number of distinct colours represented at v.Clearly,we always have $c(v)\leq d(v)\rightarrow(2)$.Moreover,$\ell$ is a proper k-edge colouring if and only if equality holds in (2) for all vertices in G.

Improvement:

We shall call a k-edge colouring ℓ' an improvement on ℓ if $\sum_{v\in V}c'(v)>\sum_{v\in V}c(v)$ where c'(v) is the number of distinct colours represented at v in the colouring ℓ'.

Optimal k-edge colouring:

An optimal k-edge colouring is one which cannot be improved.

Theorem 1.35:

If G is bipartite,then $\chi'(G)=\Delta(G)$.

Theorem 1.36 (Vizing 1964):

If G is simple,then $\chi'(G)=\Delta(G)$ or $\chi'(G)=\Delta(G)+1$.

Uniquely k-edge colourable:

A graph G is called uniquely k-edge colourable if any two proper k-edge colourings of G induce the same partition of E.

APPLICATIONS OF EDGE COLOURINGS

The Timetabling Problem:

In a school there are m teachers $X_1,X_2,...,X_m$ and n classes $Y_1,Y_2,...,Y_n$.Given the teacher X_i is required to teach class Y_j for p_{ij} periods,schedule a complete timetable in the minimum possible number of periods.This is known as the Timetabling Problem.We represent the teaching requirements by a bipartite graph G with bipartition (X,Y),where $X=\{x_1,x_2,...,x_n\}$ and $Y=\{y_1,y_2,..., y_n\}$ and vertices x_i and y_j are joined by p_{ij} edges.Now in any one period,each teacher can teach at most one class,and each class can be taught by at most one teacher.This is at least our assumption.Thus a teaching schedule for one period corresponds to a matching in the graph and,conversely,each matching corresponds to a possible assignment of teachers to classes for one period.Our problem,therefore is to partition the edges of G into as few matchings as possible or,equivalently,to properly colour the edges of G with as few colours as possible.Since G is bipartite,we know by theorem 1.35 $\chi'(G)=\Delta(G)$.Hence,if no teacher teaches for more than p periods,the teaching requirements can be scheduled in a p-period timetable.We thus have a complete solution to the problem.

1.18 PLANAR GRAPHS

Planar graph:

A graph which can be drawn in a plane so that its edges intersect only at their end vertices is called a planar graph.Such a graph is said to be embeddable in a plane.

Planar embedding:

A drawing of a planar graph G is called a planar embedding of G.

Plane graph:

A planar embedding of a planar graph is called a plane graph.

Jordan curve:

A Jordan curve is a continuous non self intersecting curve whose origin and terminus collide.

Faces:

A plane graph G partitions the rest of the plane into a number of connected regions.The closures of these regions are called the faces of

G.Each plane graph has exactly one unbounded face,called the exterior face.The degree of a face of f,$d_G(f)$,is the number of edges with which it is incident,cut edges being counted twice.

Dual graphs:

Given a plane graph G,one can define another graph G* as follows: Corresponding to each face f of G there is a vertex f* of G*,and corresponding to each edge e of G there is an edge e* of G*;Two vertices f* and g* are joined by an edge e* if and only if their corresponding faces f and g are separated by the edge e in G.The graph G* is called the dual of the graph G.

Self dual:

A plane graph is self dual if and only if it is isomorphic to its dual.

Plane triangulation:

A plane triangulation is a plane graph in which each face has degree three.

Thickness:

The minimum number of planar subgraphs whose union is the given graph G is called the thickness θ(G) of G.The thickness of a planar graph is one.The thickness of Kuratowski's graphs (K_5 and $K_{3,3}$) is two.

Crossing number:

The crossing number of a graph G is the minimum number of pair wise intersections when G is drawn in the plane.The crossing number of a planar graph is zero.The crossing number of Kuratowski's graphs (K_5 and $K_{3,3}$) is one.

Outerplanar:

A planar graph is called outerplanar if it can be embedded in the plane so that all its vertices lie on the same face.This face is often chosen to be the exterior face.A graph G is outerplanar if and only if its blocks are outerplanar.

Maximal outerplanar:

An outerplanar graph is called maximal outerplanar if no edge can be added without losing outerplanarity.Every maximal outerplanar graph is a triangulation of a polygon,while every maximal plane graph is a triangulation of the sphere.

Genus:

The genus of a graph G is defined to be the minimum number of handles to be attached to a sphere so that G can be drawn on the resulting surface without intersecting edges.Every planar graph has genus zero.K_5,K_6,K_7,$K_{3,3}$ and $K_{4,4}$ have genus one.

Theorem 1.37 (Euler):

If G is a connected plane graph,having V,E and F as the sets of vertices,edges and faces respectively,then $|V|-|E|+|F|=2$.

Corollary 1.37.1:

If G is a simple planar graph with $p\geq3$,then $q\leq3p-6$.

Corollary 1.37.2:

K_5 is nonplanar.

Corollary 1.37.3:

$K_{3,3}$ is nonplanar.

Homeomorphism:

Two graphs are called homeomorphic (or isomorphic to within vertices of degree 2) if both can be obtained from the same graph by a sequence of subdivision of the edges.

Theorem 1.38 (Kuratowski 1930):

A graph is planar,iff it has no subgraph homeomorphic to K_5 or $K_{3,3}$.

Theorem 1.39 (Five Colour Theorem):

Every planar graph is 5-vertex colourable.

APPLICATIONS OF PLANAR GRAPHS

A Planarity Algorithm:

There are many practical situations in which it is important to decide whether a given graph is planar,and,if so,then find a planar embedding of the graph.For example,in the layout of printed circuits one is interested in knowing if a particular electrical network is planar.This problem can be solved by using an algorithm known as the Planarity algorithm due to Demoucron,Malgrange,and Pertuiset (1964).

1.19 DIRECTED GRAPHS

Directed graph:

A directed graph (digraph) D is a pair (V,A),where V is a finite nonempty set and A is a subset of VxV-{(x,x)/xϵV}.The elements of V and A are respectively called vertices and arcs.If a is an arc,and u and v are vertices such that (u,v)ϵA,then a is said to join u and v.u is the tail (initial vertex) of A and v is the head (terminal vertex) of A.

Indegree $d_D^-(v)$:

The indegree $d_D^-(v)$ of a vertex v in D is the number of arcs with head v.

Outdegree $d_D^+(v)$:

The outdegree $d_D^+(v)$ of a vertex v in D is the number of arcs with tail v.

We denote the minimum and maximum indegrees and outdegrees in D by $\delta^-(D)$,$\delta^+(D)$,$\Delta^-(D)$ and $\Delta^+(D)$ respectively.

Strict digraph:

A digraph is strict if it has no loops and no two arcs with the same ends have the same orientation.

Directed walk:

A directed walk in a digraph D is a finite sequence $W=v_0a_1v_1a_2v_2\ldots v_{k-1}a_kv_k$ whose terms are alternatively vertices and arcs such that each arc a_i is incident with tail v_{i-1} and head v_i.

Directed trail:

A directed trail is a directed walk that is a trail.

Directed paths,directed cycles and directed tours are similarly defined.If there is a directed (u,v) path in D,vertex v is said to be reachable from vertex u in D.A digraph D is unilateral if,for any two vertices u and v,either v is reachable from u or u is reachable from v.

Subdigraph:

A digraph D' of D having all the vertices and arcs of D is called a subdigraph.

Underlying graph:

With each digraph D we can associate a graph G on the same vertex set.Corresponding to each arc of D there is an edge of G with the same end vertices.This graph is the underlying graph of D.

Orientation:

Given any graph G,we can obtain a digraph from G by specifying,for each link,an order on its end vertices.Such a digraph is called an orientation of G.

Theorem 1.40:

A digraph D contains a directed path of length χ-1.

Directed Hamilton path:

A path that includes every vertex of D is called a directed Hamilton path.

In-neighbour $N_D^-(v)$:

An in-neighbour $N_D^-(v)$ of a vertex v in D is a vertex u such that $(u,v) \epsilon A$.

Out-neighbour $N_D^+(v)$:

An out-neighbour $N_D^+(v)$ of a vertex v in D is a vertex w such that $(v,w) \epsilon A$.

Tournament:

A digraph D is called a tournament if for every pair of vertices u and v in D there is exactly one arc between u and v.

Score:

The score of a point in a tournament is its outdegree.

Corollary 1.41(Redei 1934):

Every tournament has a directed Hamilton path.

Directed Hamilton cycle:

A cycle that includes every vertex of D is called a directed Hamilton cycle.

Theorem 1.42:

Each vertex of a directed tournament D with $p\geq 3$ is contained in a directed k-cycle,$3\leq k\leq p$.

Theorem 1.43:

If D is strict and $\min\{\delta^{-},\delta^{+}\}\geq p/2>1$,then D contains a directed Hamilton cycle.

Directed Euler tour:

A directed Euler tour of D is a directed tour that traverses each arc of D exactly once.

k-arc connected:

A nontrivial digraph D is k-arc connected if,for every nonempty proper subset S of V,$I(S,S^{c})I\geq k$.

Associated digraph D(G):

The associated digraph D(G) of a graph G is the digraph obtained when each edge e of G is replaced by two oppositely oriented arcs with the same end vertices as e.

Strongly connected:

A digraph is called strongly connected or diconnected or strong if every pair of vertices is mutually reachable.

Unilaterally connected:

A digraph is called unilaterally connected if for every pair of vertices,at least one is reachable from the other.

Weakly connected:

A digraph is called weakly connected or weak,if the underlying graph is connected.

Disconnected:

A digraph is called disconnected if the underlying graph is disconnected.

APPLICATIONS OF DIRECTED GRAPHS

Making A Road System One-way:

Given a road system,how can it be converted to one-way operation so that traffic may flow as smoothly as possible?This is clearly a problem on orientation of graphs.

Ranking The Participants In A Tournament:

A number of players each play one another in a tennis tournament.Given the outcomes of the games,how should the participants be ranked?A possible approach would be to compute the scores (number of games won by each player) and compare them.

1.20 APPLICATIONS OF GRAPH THEORY

Graph Theory is now a major tool in mathematical research, electrical engineering,computer programming and networking,business administration,sociology,economics, marketing and communications;the list can go on and on.Inparticular,many problems can be modelled with paths formed by travelling along the edges of a certain graph.For instance, problems of efficiently planning routes for mail delivery,garbage pickup, snow removal,diagnostics in computer networks,and others,can be solved using models that involve paths in graphs.Graph Theory also has been independently discovered many times through some puzzles that arose from the physical world,consideration of chemical isomers,electrical networks etc.In our everyday life,the colourful rangolis (kolams) that we draw in front of our homes,also involve the application of graphs and their useful properties.

Chapter 2 LITERATURE REVIEW

According **to S. T. Hedetniemi, R. C. Laskar**, they divide the contributions in topics on domination theory into three sections, entitled **theoretical, new models** and **algorithmic**.

1) The nine theoretical papers retain a primary focus on properties of the standard domination number.
2) The four papers which they classify as new models are concerned primarily with new variations in the domination theme.
3) The eight algorithmic papers are primarily concerned with finding classes of graphs for which the domination number changes, and
4) Several other domination-related parameters can be computed in polynomial time.

2.1 THEORITICAL

For a variety of reasons they lead of this volume with the paper "*Chessboard domination problem*" by **Cockayne**, because he has done the most definitive work in this area.The follow up paper "*On the queen domination problem*" by **Ginstead, Hahne** and **Vanstone** is the best approximation to the old problem of placing a minimum number of queens on an arbitrary **nxn** chessboard so that all squares are covered by atleast one queen.

It is able to present next a reprint of a paper by **Berge** and **Duchet** entitled "*Recent problems and results about Kernels in directed graph*".**Claude Berge** has done more than anyone in particular of domination theory.He used the terminology **coefficient of external stability** instead of **domination number**.

David Sumner was one of the early researchers in domination theory and was perhaps the first one to consider the question of domination in critical graphs. In this paper " *Critical concepts in domination*" he considers the problem of characterizing graphs for which adding any edge e decreases the domination number.He also considers the problem of characterizing graphs having minimum dominating sets D which are independent (i.e.) no two vertices in D are adjacent.

A related notion by **Fink , Jacobson, Kinch and Roberts** in "*The bondage number of graph*", is that of finding a set of edges F of smallest order (called the bondage number), whose removal increases the domination number.

In the original survey paper on domination, **Cockayne and Hedetniemi** introduced the concept, **domatic number** of a graph denoted by **d(G)** which equals the maximum order of a partition $\{V_1, V_2, V_3, \ldots, V_R\}$ of V(G) such that every set V_i is a dominating set.Today **Zelinka** has become the world's foremost authority on domatic number and a related partition of numbers.He has published nearly two dozen papers on this topic.**Zelinka** entitled "*Regular totally domatically full graphs*" and **Rall** entitled "*Domatically critical and domatically full graphs*" on the domatic number of a graph.

2.2 NEW MODELS

The concepts of domination,covering and centrality are very well interrelated.In a 1985 paper, **Hedetniemi and Laskar** list 30 different types of domination.The paper "*Dominating cliques in graphs*" by **Cozzens and Kelleher**, studies the existence of families of graphs which contain a complete subgraph whose vertices form a dominating set.They present several forbidden subgraph conditions which are sufficient to imply the existence of dominating cliques and they present a polynomial algorithm for finding a domination clique for a certain class of graphs.

The paper "*Covering all cliques of a graph*" by **Tuza** considers a different kind of domination, in which one seeks a minimum set of vertices which dominates all cliques (i.e. maximal complete subgraphs) of a graph.

The paper by **Brigham and Dutton** entitled " *Factor domination in graphs*" considers, the general problem of finding a minimum set of vertices which is a dominating set of every subgraph in a set of edge disjoint subgraphs, say $G_1, G_2, G_3, \ldots, G_t$, whose union is a given graph G.

The paper by **Sampathkumar** entitled "*The least point covering and domination number of a graph*" is one of many papers in which one imposes additional conditions on a dominating set, e.g. the dominating set must induce a connected subgraph (connected domination), a complete subgraph (dominating clique), or a totally-disconnected graph (independent domination).In Sampathkumar's paper the domination number of the subgraph induced by the dominating set must be minimized.

2.3 ALGORITHMIC

Nearly 100 papers containing domination algorithm or complexity results have been published in the last 10 years.Perhaps, the first domination algorithm was an attempt by **Daykin** in 1966 to compute the domination number of an arbitrary tree.But his algorithm seems to have an error that cannot be easily corrected.

Cockayne, Goodman and Hedetniemi apparently constructed the first domination algorithm for trees in 1975 and about the same time, **David Johnson** constructed the first (unpublished) proof that, the domination problem for arbitrary graphs is NP complete.

The first paper by **Corneil and Stewart** entitled "*Dominating sets in perfect graphs*" presents both a brief survey of algorithmic results on domination and a discussion of the dynamic programming style technique that is commonly used in designing domination algorithms, especially as they are applied to the family of perfect graphs.

The paper "*Unit disk graphs*" by **Clark, Colbourn and Johnson** discusses the algorithmic compelxity of such problem as domination, independent domination and connected domination, and several other problems, on the intersection graphs of equal size circles in the plane.This paper is significant since it contains the result that, the domination problem for grid graphs, a subclass of unit disk graphs, is NP-complete.

In the paper "*Permutation graphs: Connected domination and Steiner trees*" by **Colbourn and Stewart**, a variety of NP-complete problems have been shown to have polynomial solutions when restricted to permutation graph.

The paper "*The discipline number of a graph*" **Chavatal and Cook**, provides an example of the relatively recent study of fractional (i.e.) real valued parameters of graphs.These are the values obtained by real relaxations of the integer linear programs corresponding to various graphical parameters like domination, matching, covering and independence.

The paper "*Best location of service centers in a tree-like network under budget constraints*" by **McHugh and Perl**, provides both nice applications perspective on domination as well as illustrations of many papers that have been published on the topic of centrality in graphs.It also provides an example of pseudo-polynomial domination algorithm an example of dynamic programming technique applied to domination problems.

The paper "*Dominating cycles in Halin graphs*" by **Skowronska and Syslo**, discusses both a fourth class of graphs on which polynomial time domination algorithms can be constructed, and the notion of a dominating cycle, (i.e.) a cycle C in a graph such that every vertex not in C lies at most one from some vertex in C.

The paper "*Finding dominating cliques efficiently***,** *in strongly chrodal graphs and undirected path graph*" by **Kratsch** is an algorithmic mate of the paper by **Cozzens and Kelleher** on dominating cliques, find the dominating cliques of minimum size.

The paper "*On minimum dominating sets with minimum intersection*" by **Grinstead and Slatter**,is a good representative of the fast developing area of polynomial, and even linear algorithms on partial K-trees.**Grinstead** and **Slatter** introduce a difficult, new type of problem, prove that it is in general NP-complete, and give a linear time solution when restricted to trees.This solution also uses the dynamic programming style approach and a methodology created by **Wimer** in his 1987 Ph.D. Thesis.

2.4 PLAN OF WORK

My dissertation starts from Chapter 3, in which the concepts of domination number, its parameters namely independent domination number,total domination number,connected domination number and edge domination number have been explained with elementary examples,ideas and some basic theorems.In Chapter 4, inverse domination number,its parameters namely inverse independent domination number,inverse total domination number,inverse connected domination number and inverse edge domination number have been explained with basic examples and theorems, corollaries and remarks.Besides, all these parameter values for some graphs like K_p,P_p,C_p,W_p and $K_{m,n}$ have been calculated.

Chapter 3 DOMINATION THEORY

3.1 HISTORY OF DOMINATION THEORY

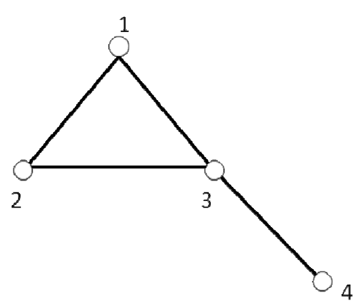
The mathematical study of dominating sets in graphs dates back to 1850's with the study of the problem of determining the minimum number of queens which are necessary to cover an nxn chessboard.More than 50 types of domination parameters have been studied by different authors.**Acharya B.D., Sampathkumar E., and Waliker H.B.** are some Indian Mathematicians who have made substantial contribution to the study of domination in graphs.In 1979 they published a technical report "*Recent developments in the theory of domination in graphs,Technical Report 14 MRI*" which triggered a considerable amount of research in this area.

3.2 DOMINATING SETS

Dominating set:

A set D of vertices in a graph G,is a dominating set,if every vertex not in D (i.e.) (every vertex in V-D) is adjacent to atleast one vertex in D.

Example:



$D_1=\{1\}$,$D_2=\{1,2\}$,$D_3=\{3\}$,$D_4=\{4,1\}$,$D_5=\{1,2,3,4\}$.

D_1 and D_2 are not dominating sets;D_3,D_4,D_5 are dominating sets.

Minimal dominating set:

A dominating set D is called a minimal dominating set,if for every vertex v,D-{v} is not a dominating set.

Minimum dominating set:

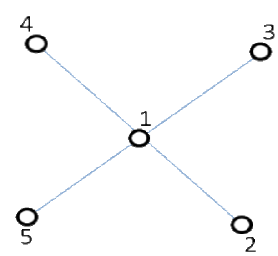
A dominating set D is called a minimum dominating set,if D consists of minimum number of vertices among all dominating sets.

3.3 DOMINATION NUMBER

Domination number $\gamma(G)$:

The number of vertices in a minimum dominating set is defined as the domination number of a graph G,and it is denoted by $\gamma(G)$.

Example:



D_1={1},D_2={2,3,4,5}.

D_1 and D_2 are minimal dominating sets.

D_1={1} is the minimum dominating set since it has minimum number of vertices than D_2.

$\gamma(G)=1$ is the domination number of G.

Observation:

For a complete graph K_p with p vertices,$\gamma(K_p)=1$.

For a path P_p with p vertices,$\gamma(P_p)=\lceil p/3 \rceil$.

For a cycle C_p with p vertices,$\gamma(C_p)=\lceil p/3 \rceil$.

For a wheel W_p with p vertices,$\gamma(W_p)=1$.

For a complete bipartite graph $K_{m,n}$ $\gamma(K_{m,n})= \min(m,n)$.

Theorem 3.31[**Ore O.,***Theory of Graphs,Amer.Math.Soc.Colloq.Publ.,38, Providence,* (1962).]:

A dominating set D is minimal dominating set if and only if for each vertex v in D,one of the following conditions holds:v is an isolated vertex of D;There exists a vertex u in V-D such that intersection of N(u) and D={v}.

Theorem 3.32[**Ore O.,***Theory of Graphs,Amer.Math.Soc.Colloq.Publ.,38, Providence,* (1962).]:

Let G be a graph without isolated vertices.If D is a minimal dominating set,then V-D is a dominating set.

Theorem 3.33:

Every superset of a dominating set is a dominating set.

Theorem 3.34:

Every minimum dominating set of a graph is a minimal dominating set.The converse of this result is not true.

Let <D> be the induced subgraph induced by the vertices of D.

3.4 INDEPENDENT DOMINATION NUMBER

Independent dominating set:

A dominating set D of a graph G is an independent dominating set,if the induced subgraph <D> has no edges.

Minimal independent dominating set:

An independent dominating set D is called a minimal independent dominating set, if for every vertex v,D-{v} is not an independent dominating set.

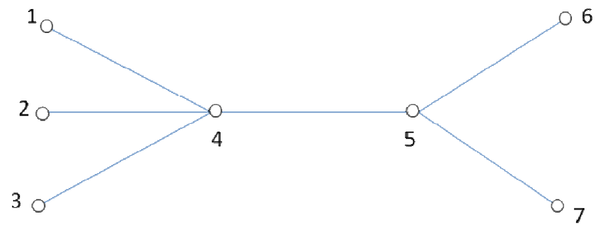
Minimum independent dominating set:

An independent dominating set D is called a minimum independent dominating set,if D consists of minimum number of vertices among all independent dominating sets.

Independent domination number $\gamma_i(G)$:

The number of vertices in a minimum independent dominating set is defined as the independent domination number of a graph G,and it is denoted by $\gamma_i(G)$.

Example:



D_1={1,2,3,5},D_2={4,6,7} are independent dominating sets.

D_3={1,2,3,6,7} is a minimal independent dominating set.

D_2={4,6,7} is the minimum independent dominating set since it has minimum number of vertices than D_1.

$\gamma_i(G)=3$ is the independent domination number of G.

Observation:

For a complete graph K_p with p vertices,$\gamma_i(K_p)=1$.

For a path P_p with p vertices,$\gamma_i(P_p)=\lceil p/3 \rceil$.

For a cycle C_p with p vertices,$\gamma_i(C_p)=\lceil p/3 \rceil$.

For a wheel W_p with p vertices,$\gamma_i(W_p)=\lceil p-1/3 \rceil$.

For a complete bipartite graph $K_{m,n}\gamma_i(K_{m,n})=\min(m,n)$.

Theorem 3.41[**Favaron O.**, *A bound on the independent domination number of a tree.Vishwa Internat.J.Graph Theory,1* (1992) 19-27.]:

For any tree T with p≥2 vertices,$\gamma_i(T)\leq(p+e)/3$.

3.5 TOTAL DOMINATION NUMBER

Total dominating set:

A dominating set D of a graph G is a total dominating set,if the induced subgraph <D> has no isolated vertices.

Minimal total dominating set:

A total dominating set D is called a minimal total dominating set,if for every vertex v,D-{v} is not a total dominating set.

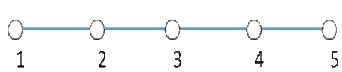
Minimum total dominating set:

A total dominating set D is called a minimum total dominating set,if D consists of minimum number of vertices among all total dominating sets.

Total domination number $\gamma_t(G)$:

The number of vertices in a minimum total dominating set is defined as the total domination number of a graph G,and it is denoted by $\gamma_t(G)$.This concept was introduced by **Cockayne, Dawes** and **Hedetniemi** in "*Total domination in graphs,Networks,10* (1980) 211-219".

Example:



D_1={1,2,3,4,5} is a total dominating set.

D_2={1,2,4,5} is a minimal total dominating set.

D_3={2,3,4} is a minimum total dominating set.

$\gamma_t(G)=3$ is the total domination number of G.

Observation:

For a complete graph K_p with p vertices, $\gamma_t(K_p)=2$.

For a path P_p with p vertices, $\gamma_t(P_p)=\lceil p/2 \rceil$ if p=4n or 4n-1.

$\lceil p+1/2 \rceil$ otherwise.

For a cycle C_p with p vertices, $\gamma_t(C_p)=\lceil p/2 \rceil$ if p=4n or 4n-1.

$\lceil p+1/2 \rceil$ otherwise.

For a wheel W_p with p vertices, $\gamma_t(W_p)=2$.

For a complete bipartite graph $K_{m,n}$ $\gamma_t(K_{m,n})=2$.

Theorem 3.51:

For any graph G, $\gamma_t(G)\geq 2$.

Theorem 3.52:

For any graph G, $\gamma(G)\leq\gamma_t(G)$.

Theorem 3.53:

If G has p vertices and no isolated vertices, then $\gamma_t(G)\leq p-\Delta(G)+1$. If G is connected and $\Delta(G)<p-1$, then $\gamma_t(G)\leq p-\Delta(G)$.

Theorem 3.54[**Cockayne E.J., Dawes R.M.** and **Hedetniemei S.T.,** *Total domination in graphs,Networks 10* (1980) 211-219.]:

If G is a connected graph with p≥3 vertices,then $\gamma_t(G) \leq (2p/3)$ and this bound is best possible.

3.6 CONNECTED DOMINATION NUMBER

Connected dominating set:

A dominating set D of a graph G is a connected dominating set,if the induced subgraph <D> is connected.

Minimal connected dominating set:

A connected dominating set D is called a minimal connected dominating set,if for every vertex v,D-{v} is not a connected dominating set.

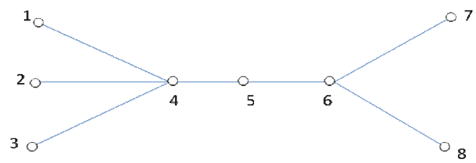
Minimum connected dominating set:

A connected dominating set D is called a minimum connected dominating set,if D consists of minimum number of vertices among all connected dominating sets.

Connected domination number $\gamma_c(G)$:

The number of vertices in a minimum connected dominating set is defined as the connected domination number of a graph G,and it is denoted by $\gamma_c(G)$.This concept was introduced by **Sampathkumar E.** and **Walikar H.B.** in "*The Connected domination number of a graph,J.Math.Phys.Sci.,13* (1979) 607-613".

Example:



D_1={4,5,6} is a connected dominating set.

D_2={4,5} is a minimal connected dominating set.

D_1={4,5,6} is a minimum connected dominating set.

$\gamma_c(G)=3$ is the connected domination number of G.

Observation:

For a complete graph K_p with p vertices,$\gamma_c(K_p)=1$.

For a path P_p with p vertices,$\gamma_c(P_p)=p-2$ if $p\geq 3$.

1 otherwise.

For a cycle C_p with p vertices,$\gamma_c(C_p)=p-2$.

For a wheel W_p with p vertices,$\gamma_c(W_p)=1$.

For a complete bipartite graph $K_{m,n}\gamma_c(K_{m,n})=\min(m,n)$.

Theorem 3.61[**Sampathkumar E.** and **Walikar H.B.,** *The connected domination number of a graph, Math.Phys.Sci.,13* (1979) 607-613.]:

If H is a connected spanning subgraph of G,then $\gamma_c(G)\leq \gamma_c(H)$.

Corollary 3.62[**Sampathkumar E.** and **Walikar H.B.,** *The connected domination number of a graph, Math.Phys.Sci.,13* (1979) 607-613.]:

For any connected graph G with $p\geq 3$ vertices,$\gamma_c(G)\leq p-2$.

Corollary 3.63:

If T is a tree and $p\geq3$,then $\gamma_c(T)=p-e$ where e denotes the number of end vertices of a tree T.

Theorem 3.64[**Hedetniemi S.T.** and **Laskar R.C.,***Connected domination in graphs,*In **Bollobas B.,**editor,*Graph Theory and Combinatorics, Academic Press, London* (1984) 209-218.]:

For any connected graph G,$\gamma_c(G)\leq p-\Delta(G)$.

Corollary 3.65:

For any tree T,$\gamma_c(T)=p-\Delta(T)$ if and only if T has atmost one vertex of degree three or more.

3.7 EDGE DOMINATION NUMBER

Edge dominating set:

A set F of edges in a graph G,is an edge dominating set,if every edge not in F (i.e.) (every edge in E-F) is adjacent to at least one edge in F.

Minimal edge dominating set:

An edge dominating set F is called a minimal edge dominating set,if for every edge e,F-{e} is not an edge dominating set.

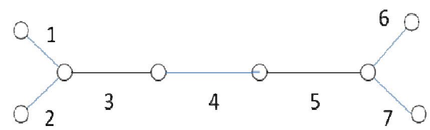
Minimum edge dominating set:

An edge dominating set F is called a minimum edge dominating set,if F consists of minimum number of edges among all edge dominating sets.

Edge domination number $\gamma_e(G)$:

The number of edges in a minimum edge dominating set is defined as the edge domination number of a graph G,and it is denoted by $\gamma_e(G)$.

Example:



$F_1=\{1,2,3,5\}$ is an edge dominating set.

$F_2=\{1,4,6\}$ is a minimal edge dominating set.

$F_3=\{1,5\}$ is a minimum edge dominating set.

$\gamma_e(G)=2$ is the edge domination number of G.

Observation:

For a complete graph K_p with p vertices,$\gamma_e(K_p)=\lfloor p/2 \rfloor$.

For a path P_p with p vertices,$\gamma_e(P_p)=\lfloor p+1/3 \rfloor$.

For a cycle C_p with p vertices,$\gamma_e(C_p)=\lceil p/3 \rceil$.

For a wheel W_p with p vertices,$\gamma_e(W_p)=\lceil p/3 \rceil$.

For a complete bipartite graph $K_{m,n}\gamma_e(K_{m,n})=\min(m,n)$.

Theorem 3.71[**Jayaram S.R**, *Line domination in graphs,Graphs Combin.3* (1987) 357-363.]:

An edge dominating set F is minimal if and only if for each edge eϵF,one of the following two conditions hold.

The intersection of N(e) and F is empty.

There exists an edge $e_1 \epsilon E-F$ such that the intersection of $N(e_1)$ and F is {e}.

Theorem 3.72[**Jayaram S.R**, *Line domination in graphs,Graphs Combin.3* (1987) 357-363.]:

Let G be a graph without isolated edges.If F is a minimal edge dominating set, then E-F is an edge dominating set.

Theorem 3.73:

Every superset of an edge dominating set is a dominating set.

Theorem 3.74[**Jayaram S.R,** *Line domination in graphs,Graphs Combin.3* (1987) 357-363.]:

Let Δ'(G) denotes the maximum degree among the edges of a graph G.Then $\gamma_e(G) \leq q-\Delta'(G)$.

Theorem 3.75[**Jayaram S.R**, *Line domination in graphs,Graphs Combin.3* (1987) 357-363.]:

If G is a (p,q) graph without isolated vertices,then $q/\Delta'(G)+1 \leq \gamma_e(G)$.

Chapter 4 INVERSE DOMINATION THEORY

4.1 INVERSE DOMINATION NUMBER

Inverse dominating set:

Let D be a minimum dominating set in a graph G.If V-D contains a dominating set D'of G,then D' is called an inverse dominating set with respect to D.

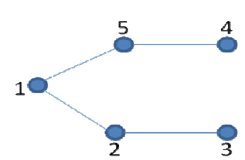
Minimum inverse dominating set:

An inverse dominating set D is called a minimum inverse dominating set,if D consists of minimum number of vertices among all inverse dominating sets.

Inverse domination number $\gamma^{-1}(G)$:

The number of vertices in a minimum inverse dominating set is defined as the inverse domination number of a graph G,and it is denoted by $\gamma^{-1}(G)$.This concept was introduced by **Kulli V.R.** and **Sigarkanti S.C.** in "*Inverse domination in graphs,Nat. Acad.Sci.Lett.,14*(1991) 473-475".

Example:



D_1={2,5},D_2={2,4},D_3={3,5} are the minimum dominating sets.Their corresponding inverse dominating sets are D_1*={1,3,4}, D_2*={3,5}, D_3*={2,4} respectively.Thus $\gamma(G)=2$ is the domination number of G.

$\gamma^{-1}(G)=2$ is the inverse domination number of G.

Note:

Every graph without isolated vertices contains an inverse dominating set, since the complement of any minimal dominating set is also a dominating set.Thus we consider a graph without isolated vertices.

Observation:

For a complete graph K_p with p vertices,$\gamma^{-1}(K_p)=1$.

For a path P_p with p vertices,$\gamma^{-1}(P_p)=\lceil p+1/3 \rceil$.

For a cycle C_p with p vertices,$\gamma^{-1}(C_p)=\lceil p/3 \rceil$.

For a wheel W_p with p vertices,$\gamma^{-1}(W_p)=\lceil p-1/3 \rceil$.

For a complete bipartite graph $K_{m,n}\gamma^{-1}(K_{m,n})=p-1$ if $m=1$ or $n=1$,2 if $m\geq 2$ or $n\geq 2$.

Proposition 4.11:

If a graph G has no isolated vertices,then $\gamma(G)\leq\gamma^{-1}(G)$.

Proposition 4.12:

If a graph G has no isolated vertices,then $\gamma(G)+\gamma^{-1}(G)\leq p$.

Theorem 4.13[**Kulli V.R.** and **Sigarkanti S.C.,** *Inverse domination in graphs, Nat.Acad.Sci.Lett.,14* (1991) 473-475.]:

Let D be a minimum dominating set of G.If for every vertex v in D,the induced subgraph <N[v]> is a complete graph of order at least 2, then

$\gamma(G)=\gamma^{-1}(G)$.

Theorem 4.14[**Kulli V.R.** and **Sigarkanti S.C.,** *Inverse domination in graphs, Nat.Acad.Sci.Lett.,14* (1991) 473-475.]:

Let $\boldsymbol{\tau}$ denote the family of minimum dominating sets of G.If every minimum dominating set $D\epsilon\boldsymbol{\tau}$,V-D is independent,then $\gamma(G)+\gamma^{-1}(G)=p$.

Theorem 4.15[**Kulli V.R.** and **Sigarkanti S.C.,** *Inverse domination in graphs, Nat.Acad.Sci.Lett.,14* (1991) 473-475.]:

If every non end vertex of a tree T is adjacent to atleast one end vertex,then $\gamma(T)+\gamma^{-1}(T)=p$.

Theorem 4.16[**Kulli V.R.** and **Sigarkanti S.C.,** *Inverse domination in graphs, Nat.Acad.Sci.Lett.,14* (1991) 473-475.]:

If G is a (p,q) graph with $\gamma(G)=\gamma^{-1}(G)$,then $(2p-q)/3\leq\gamma^{-1}(G)$.

Theorem 4.17:

If a (p,q) graph G has no isolated vertices,then $(2p-q)/3\leq\gamma^{-1}(G)$.

Corollary 4.18[**Domke G.S., DunbarJ.E**. and **MarkusL.R,** *The Inverse domination number of a graph,Ars.Combin.,72* (2004) 149-160.]:

For any tree T of order $p\geq2,(p+1/3)\leq\gamma^{-1}(T)$.

The Kulli-Sigarkanti Conjecture 4.19[**Hedetniemi S.T., LaskarR.C., Markus L**. and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

For any graph G without isolated vertices,$\gamma^{-1}(G)\leq\beta(G)$.

Remark[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

The disjoint domination number $\gamma\gamma(G)$ of a graph G is the minimum cardinality of the union of two disjoint dominating sets in G.

Theorem 4.110[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

If G is a graph of order $p\geq 2$ with no isolated vertices,then $\gamma\gamma(G)\leq\gamma^{-1}(G)\leq\beta(G)$.

Corollary 4.111[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

If G is a graph of order $p\geq 2$ with no isolated vertices and if $\gamma(G)=\gamma^{-1}(G)$, then $\gamma^{-1}(G) \leq \beta(G)$,that is the Kulli-Sigarkanti Conjecture is true when $\gamma(G)=\gamma^{-1}(G)$.

Corollary 4.112[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

The Kulli-Sigarkanti Conjecture is true when G has no isolated vertices and no induced subgraph is isomorphic to $K_{1,3}$.In particular,the Kulli-Sigarkanti Conjecture is true for line graphs.

Theorem 4.113[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

If G is a connected graph with p vertices and q edges,then $q\geq 2p-3\gamma\gamma(G)/2$.

Corollary 4.114[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

If T is a tree,then $\gamma\gamma(T) \geq 2(p+1)/3$.

Theorem 4.115[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

If G and G^c have no isolated vertices and diam(G)≥4then $\gamma\gamma(G)+ \gamma\gamma(G^c) \leq p+4$.

4.2 INVERSE INDEPENDENT DOMINATION NUMBER

Inverse independent dominating set:

Let D be a minimum independent dominating set in a graph G.If V-D contains an independent dominating set D'of G,then D' is called an inverse independent dominating set with respect to D.

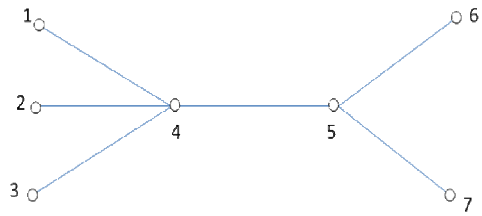
Minimum inverse independent dominating set:

An inverse independent dominating set D is called a minimum inverse independent dominating set,if D consists of minimum number of vertices among all inverse independent dominating sets.

Inverse independent domination number $\gamma_i^{-1}(G)$:

The number of vertices in a minimum inverse independent dominating set is defined as the inverse independent domination number of a graph G,and it is denoted by $\gamma_i^{-1}(G)$.This concept was introduced by **Kulli V.R.** and **Nandargi N.R.** in "*Inverse domination and some parameters,manuscript*".

Example:



D_1={4,6,7} is the minimum independent dominating set.Its corresponding inverse independent dominating set is D_1*={1,2,3,5}.Thus $\gamma_i(G)=3$ is the independent domination number of G.$\gamma_i^{-1}(G)=4$ is the inverse independent domination number of G.

Note:

Every graph without isolated vertices contains an inverse independent dominating set.Thus we consider only graphs without isolated vertices.

Observation[**Kulli V.R.** and **Nandargi N.R.** in *Inverse domination and some parameters,manuscript.*]:

For a complete graph K_p with p vertices,$\gamma_i^{-1}(K_p)=1$.

For a path P_p with p vertices,$\gamma_i^{-1}(P_p)=\lceil p+1/3 \rceil$.

For a cycle C_p with p vertices,$\gamma_i^{-1}(C_p)=\lceil p/3 \rceil$.

For a wheel W_p with p vertices,$\gamma_i^{-1}(W_p)=\lceil p-1/3 \rceil$.

For a complete bipartite graph $K_{m,n}\gamma_i^{-1}(K_{m,n})$=p-1 if m=1 or n=1,2 if m≥2 or n≥2.

Theorem 4.21[**Kulli V.R.** and **Nandargi N.R.** in *Inverse domination and some parameters,manuscript.*]:

If a graph G has no isolated vertices, then $\gamma_i(G) \leq \gamma_i^{-1}(G)$. Furthermore,equality holds if $G=K_p,C_p,P_{3k+1},P_{3k+2},k\geq 1$.

Theorem 4.22[**Kulli V.R.** and **Nandargi N.R.** in *Inverse domination and some parameters,manuscript.*]:

If a graph G has no isolated vertices,then $\gamma_i(G)+\gamma_i^{-1}(G) \leq p$. Furthermore,equality holds if $G=K_2,C_4,P_3,P_4$.

Remark[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

The disjoint independent domination number $\gamma\gamma_i(G)$ of a graph G is the minimum cardinality of the union of two disjoint independent dominating sets in G.

Theorem 4.23[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

For any tree T of order $p \geq 2, \gamma\gamma(T)=\gamma\gamma_i(T)$.

Corollary 4.24[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

For any tree T of order $p \geq 2, \gamma\gamma(T)=\gamma\gamma_i(T) \leq \gamma_i^{-1}(T) \leq \beta(T)$.

4.3 INVERSE TOTAL DOMINATION NUMBER

Inverse total dominating set:

Let D be a minimum total dominating set in a graph G.If V-D contains a total dominating set D’of G,then D’ is called an inverse total dominating set with respect to D.

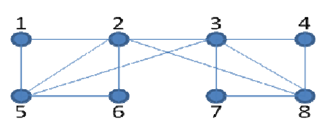
Minimum inverse total dominating set:

An inverse total dominating set D is called a minimum inverse total dominating set, if D consists of minimum number of vertices among all inverse total dominating sets.

Inverse total domination number $\gamma_t^{-1}(G)$:

The number of vertices in a minimum inverse total dominating set is defined as the inverse total domination number of a graph G, and it is denoted by $\gamma_t^{-1}(G)$.This concept was introduced by **Kulli V.R.** and **Iyer R.R.** in “*Inverse total domination in graphs,J.Discrete Mathematical Sciences and Cryptography,to appear*”.

Example:



$D_1=\{2,3\}$,$D_2=\{2,8\}$,$D_3=\{3,5\}$ are the minimum total dominating sets.Their corresponding inverse total dominating sets are $D_1^*=\{5,6,7,8\}$, $D_2^*=\{3,5\}$, $D_3^*=\{2,8\}$ respectively.Thus $\gamma_t(G)=2$ is the total domination number of G. $\gamma_t^{-1}(G)=2$ is the inverse total domination number of G.

Note:

Every graph without isolated vertices contains an inverse total dominating set.Thus we consider only graphs without isolated vertices.

Observation[**Kulli V.R.** and **Iyer R.R.** *Inverse total domination in graphs,J.Discrete Mathematical Sciences and Cryptography,to appear.*]:

For a complete graph K_p with p vertices,$\gamma_t^{-1}(K_p)=2$.

For a path P_p with p vertices,$\gamma_t^{-1}(P_p)$ does not exist since there is a vertex of degree 1.

For a cycle C_p with p vertices,$\gamma_t^{-1}(C_p)=p/2$,p is a multiple of 4,does not exist otherwise.

For a wheel W_p with p vertices,$\gamma_t^{-1}(W_p)=(p+1)/2$ if p=3(mod4),$\lceil p-1/3 \rceil$ otherwise.

For a complete bipartite graph $K_{m,n}\gamma_t^{-1}(K_{m,n})=2$ if $2\leq m\leq n$.

Proposition 4.31[**Kulli V.R.** and **Iyer R.R.** *Inverse total domination in graphs,J.Discrete Mathematical Sciences and Cryptography,to appear.*]:

Let D be a total dominating set of a connected graph G.If an inverse total dominating set exists,then G has atleast 4 vertices.

Proposition 4.32[**Kulli V.R.** and **Iyer R.R.** *Inverse total domination in graphs,J.Discrete Mathematical Sciences and Cryptography,to appear.*]:

If a graph G has an inverse total dominating set,then $\gamma_t(G)\leq\gamma_t^{-1}(G)$.

Proposition 4.33[**Kulli V.R.** and **Iyer R.R.** *Inverse total domination in graphs,J.Discrete Mathematical Sciences and Cryptography,to appear.*]:

If a graph G has an inverse total dominating set,then $\gamma_t(G)+\gamma_t^{-1}(G)\leq p$.

Proposition 4.34[**Kulli V.R.** and **Iyer R.R.** *Inverse total domination in graphs,J.Discrete Mathematical Sciences and Cryptography,to appear.*]:

If a graph G has an inverse total dominating set,then $2\leq\gamma_t^{-1}(G)\leq p-2$.

Remark[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

The disjoint total domination number $\gamma\gamma_t(G)$ of a graph G is the minimum cardinality of the union of two disjoint total dominating sets in G.

Proposition 4.35[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

If a graph G has an inverse total dominating set,then $2\gamma_t(G)\leq\gamma\gamma_t(G)\leq\gamma_t(G)+\gamma_t^{-1}(G)\leq p$.

Note[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

We say that a graph G is $\gamma\gamma_t$-minimum if it has two disjoint total dominating sets such that,$\gamma\gamma_t(G)=2\gamma_t(G)$.A graph G is $\gamma\gamma_t$-maximum if it has two disjoint total dominating sets such that,$\gamma\gamma_t(G)=p$.A graph G is called $\gamma\gamma_t$-strong if,$\gamma\gamma_t(G)=2\gamma_t(G)=p$.

4.4 INVERSE CONNECTED DOMINATION NUMBER

Inverse connected dominating set:

Let D be a minimum connected dominating set in a graph G.If V-D contains a connected dominating set D'of G,then D' is called an inverse connected dominating set with respect to D.

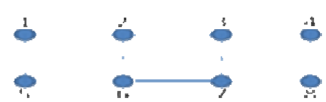
Minimum inverse connected dominating set:

An inverse connected dominating set D is called a minimum inverse connected dominating set,if D consists of minimum number of vertices among all inverse connected dominating sets.

Inverse connected domination number $\gamma_c^{-1}(G)$:

The number of vertices in a minimum inverse connected dominating set is defined as the inverse connected domination number of a graph G,and it is denoted by $\gamma_c^{-1}(G)$.

Example:



$D_1=\{2,3\}$,$D_2=\{2,8\}$,$D_3=\{3,5\}$are the minimum connected dominating sets.Their corresponding inverse connected dominating sets are $D_1^*=\{5,6,7,8\}$,$D_2^*=\{3,5\}$,$D_3^*=\{2,8\}$ respectively.Thus $\gamma_c(G)=2$ is the connected domination number of G.$\gamma_c^{-1}(G)=2$ is the inverse connected domination number of G.

Note:

Only connected nontrivial graphs have an inverse connected dominating set.

Observation:

For a complete graph K_p with p vertices,$\gamma_c^{-1}(K_p)=1$.

For a path P_p with p vertices,$\gamma_c^{-1}(P_p)$ does not exist since there is a vertex of degree 1.

For a cycle C_p with p vertices,$\gamma_c^{-1}(C_p)=2$ if $p\leq 5$,does not exist otherwise.

For a wheel W_p with p vertices,$\gamma_c^{-1}(W_p)=p-3$.

For a complete bipartite graph $K_{m,n}\gamma_c^{-1}(K_{m,n})=\min(m,n)$.

Proposition 4.41:

If a graph G has an inverse total dominating set,then $\gamma_c(G)\leq\gamma_c^{-1}(G)$ and

$\gamma_c(G)+\gamma_c^{-1}(G)\leq p$.

4.5 INVERSE EDGE DOMINATION NUMBER

Inverse edge dominating set:

Let F be a minimum edge dominatng set in a graph G.If E-F contains an edge dominating set F'of G,then F' is called an inverse edge dominating set of G with respect to F.

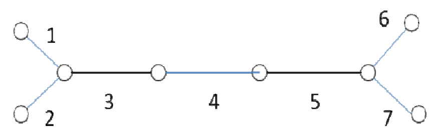
Minimum inverse edge dominating set:

An inverse edge dominating set F is called a minimum inverse edge dominating set,if F consists of minimum number of edges among all inverse edge dominating sets.

Inverse edge domination number $\gamma_e^{-1}(G)$:

The number of edges in a minimum inverse edge dominating set is defined as the inverse edge domination number of a graph G,and it is denoted by $\gamma_e^{-1}(G)$.This concept was introduced by **Kulli V.R.** and **Soner N.D.** in "*Complementary edge domination in graphs,Indian J.Pure Appl.Math.28* (1997) 917-920".

Example:



D_1={1,5} is a minimum edge dominating set.Its corresponding inverse edge dominating set is D_1^*={3,6}.Thus $\gamma_e(G)=2$ is the edge domination number of G.$\gamma_e^{-1}(G)=2$ is the inverse edge domination number of G.

Note:

Every graph without isolated edges contains an inverse edge dominating set, since the complement of any minimal edge dominating set is also an edge dominating set.Thus we consider a graph without isolated edges.

Observation[**Kulli V.R.** and **Soner N.D.**, *Complementary edge domination in graphs,Indian J.Pure Appl.Math.28* (1997) 917-920.]:

For a complete graph K_p with p vertices,$\gamma_e^{-1}(K_p)=\lfloor p/2 \rfloor$ if $p\geq3$.

For a path P_p with p vertices,$\gamma_e^{-1}(P_p)=\lceil p/3 \rceil$ if $p\geq3$.

For a cycle C_p with p vertices,$\gamma_e^{-1}(C_p)=\lceil p/3 \rceil$ if $p\geq3$.

For a wheel W_p with p vertices,$\gamma_e^{-1}(W_p)=\lceil p/3 \rceil$ if $p\geq4$.

For a complete bipartite graph $K_{m,n}\gamma_e^{-1}(K_{m,n})=\min(m,n)$.

Proposition 4.51[**Kulli V.R.** and **Soner N.D.**, *Complementary edge domination in graphs,Indian J.Pure Appl.Math.28* (1997) 917-920.]:

If a graph G has no isolated edges,then $\gamma_e(G)\leq\gamma_e^{-1}(G)$.

Proposition 4.52[**Kulli V.R.** and **Soner N.D.**, *Complementary edge domination in graphs,Indian J.Pure Appl.Math.28* (1997) 917-920.]:

If a graph G has no isolated edges,then $\gamma_e(G)+\gamma_e^{-1}(G)\leq q$.

Theorem 4.53[**Kulli V.R.** and **Soner N.D.**, *Complementary edge domination in graphs,Indian J.Pure Appl.Math.28* (1997) 917-920.]:

Let D be a minimum dominating set of G.If for every edge e in F,the induced subgraph <N[e]> is a star,then $\gamma_e(G)=\gamma_e^{-1}(G)$.

Theorem 4.54:

For any graph G without isolated vertices and isolated edges and $p\geq3$, then $\gamma_e^{-1}(G)\leq\beta'(G)$.

Remark[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

The disjoint edge domination number $\gamma\gamma_e(G)$ of a graph G is the minimum cardinality of the union of two disjoint edge dominating sets in G.

Proposition 4.55[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

If a graph G has an inverse edge dominating set,then $2\gamma_e(G)\leq\gamma\gamma_e(G)\leq\gamma_e(G)+\gamma_e^{-1}(G)\leq q$.

Note[**Hedetniemi S.T., LaskarR.C., Markus L.** and **Slater P.J.,** *Disjoint dominating sets in graphs,to appear.*]:

We say that a graph G is $\gamma\gamma_e$-minimum if $\gamma\gamma_e(G)=2\gamma_e(G)$.A graph G is $\gamma\gamma_e$-maximum if $\gamma\gamma_t(G)=q$.A graph G is called $\gamma\gamma_e$-strong if,$\gamma\gamma_e(G)=2\gamma_e(G)=q$.

CONCLUSION

Thus we have calculated the inverse domination number and the parameters of inverse domination number for graphs like K_p,P_p,C_p,W_p and $K_{m,n}$.

APPLICATIONS OF DOMINATION THEORY

Domination is an area in graph theory with an extensive research activity.The dominating set problem tells to determine the domination number of a graph.The concept of dominating set occurs in a variety of problems.A number of problems are motivated by communication network problems.The communication network includes a set of nodes,where one node can communicate with another,if it is directly connected to that node.Inorder to send a message directly from a set of nodes to all others,one has to choose the set in such a way that all other nodes are connected to atleast one node in the set.Now,such a set is a dominating set in a graph,which represents the network.Other applications of domination are the facility location problem,land surveying and routings.

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GLOSSARY OF SYMBOLS

A	arc set,52
c(G)	closure,35
$d_G(v)$	degree of vertex v in G,20
$d_G(f)$	degree of face f in G,49
$d_D^-(v)$	indegree of vertex v in D,52
$d_D^+(v)$	outdegree of vertex v in D,52
$d_G(u,v)$	distance between u and v in G,24
diam(G)	maximum distance between u and v in G,24
D	directed graph (digraph),52
D(G)	associated digraph of G,55
e(v)	eccentricity of a vertex v,32
E, E(G)	edge set,13
F	face set,50
G	graph,13
G^c	complement of G,17
G*	dual of G,49
G[V']	subgraph of G induced by V,18
G[E']	subgraph of G induced by E,18
G-v	deletion of a vertex v from G,18
G-e	deletion of an edge e from G,19

$G+e$	addition of an edge e from G,19
$G \cdot e$	contraction of an edge,19
K_n	complete graph on n vertices,31
$K_{m,n}$	complete bipartite graph on n vertices,15
P_n	path with n vertices,16
C_n	cycle with n vertices,23
M	matching,24
$N_G(S)$	neighbour set of S in G,40
$N_D^-(v)$	in-neighbour set of v in D,41
$N_D^+(v)$	out-neighbour set of v in D,54
$r(k,l)$	ramsey number,54
$V, V(G)$	vertex set,39
$\beta(G)$	independence number of G,13
$\beta'(G)$	edge-independence number of G,37
$\alpha(G)$	covering number of G,37
$\alpha'(G)$	edge-covering number of G,38
δ	minimum degree,20
δ^-	minimum indegree,52
δ^+	minimum outdegree,52
Δ	maximum degree,20
Δ^-	maximum indegree,52

Δ^+	maximum outdegree,52
o	number of odd components,42
κ	connectivity,27
κ'	edge connectivity,27
p,v	number of vertices,13
q,e	number of edges,13
r(G)	radius of G,32
f(G,t)	chromatic polynomial,45
$\chi(G)$	chromatic number of G,44
$\chi'(G)$	edge-chromatic number of G,46
τ	number of spanning trees,31
GUH	union,19
G+H	sum,19
GxH	product,19
G[H]	composition,19
$\gamma(G)$	domination number of G,65
$\gamma_i(G)$	independent domination number of G,67
$\gamma_t(G)$	total domination number of G,69
$\gamma_c(G)$	connected domination number of G,71
$\gamma_e(G)$	edge domination number of G,73
$\gamma^{-1}(G)$	inverse domination number of G,76

$\gamma_i^{-1}(G)$ inverse independent domination number of G,80

$\gamma_t^{-1}(G)$ inverse total domination number of G,83

$\gamma_c^{-1}(G)$ inverse connected domination number of G,85

$\gamma_e^{-1}(G)$ inverse edge domination number of G,87

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